A mean field approach for large-scale interacting networks of conductance-based spiking neurons: application to the evaluation of statistical estimates of certain elements of the corticobasal ganglia network

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1 Introduction.

We condider coupled networks of neural spiking cells whose dynamivcal behavior is governed, at the individual level, according to conductance-based neuronal models. The complete system is made of a set of networks. Cells in a given network have the same dynamical features but these later may vary from one network to another one. The neuronal models of the basic units enable the characterization of the states of the individual neurons and in particular provide the means to determine the times of occurence of spikes. In addition to sets of dynamical variables consisting of activation and inactivation variables for ionic channel and calcium concentration, the individual neurons are endowed with a spatial structure in the form of a dendritic tree attached to soma.

The networks we consider are large and, moreover, are subject to noisy disturbances of the white noise type. We have shown in [6] that it is possible to construct a mean field theory for such systems. In fact, in [6], the neural systems are punctual. Here we generalize the framework of this kinetic theory to neural systems with a dendritic tree structure. This mean field theory makes it possible to construct, for these coupled networks, a system of non-linear, partial integrodifferential equations for the probability distributions of all the dynamic variables of the system (membrane and dendritic potentials, ion channel and connection variables).

The gain in computing time, in numerical resolution of these systems, in the transition from a very large number of coupled stochastic equations controlling individual dynamics to a smaller number of equations for probability distributions can be important. This analysis is developed here firstly within the general framework of any number of coupled networks. Then, an example of application of this method which treats the mammalian basal ganglia system is presented. The latter is made up of up and down spiny neurons of the striatum, the internal and external globus pallidus networks, the subthalamic nucleus, the substantia nigra pars compacta and finally a group of fast spiking interneurons from the striatum. The system of integrodifferential equations for the probability distributions of this set of networks is presented. Numerical resolution is then carried out for some of its subsets. The objective aimed here is to show that modeling including a significant part of the biologically plausible characteristics of cells can be proposed in the case where the neuronal populations are of large size, this being able to be done without having recourse to the necessity, difficult to apprehend , usually encountered, of solving huge systems of coupled (stochastic) differential equations.

2 A dynamical system for coupled large-scale biologically plausible neuronal networks.

We consider coupled neworks of spiking cells whose dynamical behavior is governed, at the individual level, according to conductance-based neuronal models. The complete system is made of P networks of cells, where all neurons in a given population $\mathscr{P}_\gamma, \gamma = 1, 2..P$ have the same dynamical features but these later may vary from one population to another one. In what follows, the number of cells in $\mathscr{P}_\gamma$ is called K_γ.....

2.1 The cellular model in $\mathscr{P}_\gamma$: m_γ dimensional ionic variables, d_γ dimensional compartmental dendritic arborescence structure.

In this section, we define the neuronal models which are the basic units of the networks. Such models enable the characterization of the states of the individual neurons and in particular provide the means to determine the times of occurrence of spikes. In addition to m_γ dynamical variables consisting of activation and inactivation variables for ionic channels and calcium ion concentrations, the individual neurons are endowed with a spatial structure in the form of a dendritic tree attached to a soma. A typical form of the dendritic trees, which is ascribed a dimensionality d_γ, is shown in Fig.i. The state variables of each $cell_i$ $i = 1, 2, \ldots, K_\gamma$ in $\mathscr{P}_\gamma$ are its membrane soma potential V_i^γ, a m_γ-dimensional set of recovery gating variables $\widehat{R}_i^\gamma$ which control the ionic channels activity and a d_γ-dimensional multicompartmental vector $\widetilde{Y}_i^\gamma$ which describes the potential values in the various parts of the arborescent dendritic structure. We call $I^\gamma(V_i^\gamma, \widetilde{Y}_i^\gamma, \widehat{R}_i^\gamma)$ (resp. $\widetilde{\Xi}^\gamma(V_i^\gamma, \widetilde{Y}_i^\gamma, \widehat{R}_i^\gamma)$) the sum of ionic, active channels and passive leak, transverse (resp. transverse and longitudinal) currents across (resp. across and along) the membrane of the soma (resp. multicompartments dendritic tree) of the i^{th} cell in $\mathscr{P}_\gamma$. Moreover, the dynamical laws for variables $\widehat{R}_i^\gamma$ are defined in terms of a m_γ dimensional vector valued function $\widehat{\Psi}^\gamma(V_i^\gamma, \widetilde{Y}_i^\gamma, \widehat{R}_i^\gamma)$. The synaptic inputs on $cell_i$ which are distributed on the soma and the dendritic compartments are denoted $I_{i,syn}^{s,\gamma}(t)$ and $\widetilde{I}_{i,syn}^{d,\gamma}(t)$. External currents may also be applied on somas and dendritic compartments of all cells of the network. They are composed of a deterministic part ($I_{ext}^{s,\gamma}$ and $\widetilde{I}_{ext}^{d,\gamma}$) and a stochastic part. Finally, the dynamic system controlling the activity of the cells of the system has the following form, C_s^γ (resp. C_d^γ)being the transmembrane capacitance of the somas (resp. dendrites).

$$\frac{dV_i^\gamma}{dt} = (1/C_s^\gamma)\big\{I^\gamma(V_i^\gamma, \widetilde{Y}_i^\gamma, \widehat{R}_i^\gamma) + I_{i,syn}^{s,\gamma}(t) + I_{ext}^{s,\gamma}(t)\big\} + \eta_{i,t}^\gamma \tag{1}$$

$$\frac{d\widetilde{Y}_i^\gamma}{dt} = (1/C_d^\gamma)\big\{\widetilde{\Xi}^\gamma(V_i^\gamma, \widetilde{Y}_i^\gamma, \widehat{R}_i^\gamma) + \widetilde{I}_{i,syn}^{d,\gamma}(t) + \widetilde{I}_{ext}^{d,\gamma}(t)\big\} \tag{2}$$

$$\frac{d\widehat{R}_i^\gamma}{dt} = \widehat{\Psi}^\gamma(V_i^\gamma, \widetilde{Y}_i^\gamma, \widehat{R}_i^\gamma) \tag{3}$$

where $\eta_{i,t}^\gamma$ is a white noise such that

$$\langle \eta_{i,s}^\gamma \eta_{j,t}^\gamma \rangle = \delta_{ij}\, \delta(s-t)\, \beta_i^\gamma, \quad i, j = 1, 2, \ldots, K_\gamma, \tag{4}$$

Moreover, δ_{ij} is the Kronecker symbol, $\delta(\cdot)$ is the delta distribution and β_i^γ $i = 1, 2, \ldots, K_\gamma$ are noise parameters. We assume that the deterministic parts of the currents and the noise parameters are the same for all neurons in each given population.
In the following, the system of (stochastic) coupled differential equations (1), (2), (3) will be called SCDE.

2.2 The dendritic structure.

Let us introduce the dendritic function $\widetilde{\Xi}^\gamma(V_i^\gamma, \widetilde{Y}_i^\gamma, \widehat{R}_i^\gamma)$ which appears in (2), for an arbitrary cell, in an arbitrary population $\mathscr{P}_\gamma, \gamma = 1, 2..P$. The index i is therefore omitted. The potential at the trigger somatic zone is denoted by V^γ. The m_γ-dimensional set of recovery gating variables is denoted $\widehat{R}^\gamma$. Finally, we call $\widetilde{Y}^\gamma$ a d_γ-dimensional multicompartments dendritic potential vector. The vector $\widetilde{Y}^\gamma$ has the representation $\widetilde{Y}^\gamma = (\{\widetilde{Y}_{i_1}^\gamma\}_{i_1}, \{\widetilde{Y}_{i_1 i_2}^\gamma\}_{i_1 i_2}, ..., \{\widetilde{Y}_{i_1 i_2 \ldots i_k}^\gamma\}_{i_1 i_2 \ldots i_k}, ...), i_1 = 1...Q^1, i_2 = 1, ...Q_{i_1}^2, i_3 = 1, ...Q_{i_1 i_2}^3, ...$ The nearest (connected to the trigger zone compartment) first level compartments, are represented by $\widetilde{Y}_{i_1}^\gamma, i_1 = 1, 2, \ldots, Q^1$ where Q^1 is the number of these compartments. In Fig.i,$Q^1 = 3$. The notation $\widetilde{Y}_{i_1 i_2}^\gamma, i_2 = 1, 2, \ldots, Q_{i_1}^2$ is used for the compartments which are connected to this fist level compartments. In Fig.i, $Q_1^2 = 2, Q_2^2 = 2, Q_3^2 = 2$.

Higher branches are similarly characterized ($Q^3_{11}=3, Q^3_{12}=3, Q^3_{21}=1$, and so on). For a model with maximum number of branching levels B, the compartmental variables at this level are called $\widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_B}$. In Fig.i, $B=5$. Transmembrane conductance of compartment $i_1 i_2 \ldots i_k$ is denoted $\alpha_{i_1 i_2 \ldots i_k}$. Junctional conductances between compartments $i_1 i_2 \ldots i_k$ and $i_1 i_2 \ldots i_{k+1}$ are called $\zeta_{i_1 i_2 \ldots i_k, i_1 i_2 \ldots i_{k+1}}$.

Using these notations, for a compartment which is neither the soma nor a terminal one, we have

$$\begin{aligned}\frac{d}{dt}\widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_k} &= -\alpha_{i_1 i_2 \ldots i_k}\widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_k} + \zeta_{i_1 i_2 \ldots i_{k-1}, i_1 i_2 \ldots i_k}(\widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_{k-1}} - \widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_k}) \\ &+ \sum_{j=1}^{Q^{k+1}_{i_1 i_2 \ldots i_k}} \zeta_{i_1 i_2 \ldots i_k, i_1 i_2 \ldots i_k j}(\widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_k j} - \widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_k}) + \widetilde{I}^{ch}_{i_1 i_2 \ldots i_k}(\widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_k}, \widehat{R}^{\gamma})\end{aligned} \tag{5}$$

For the proximal compartments, xxxx.
Introduce capacitance, xxxx
For the K terminal compartments, one has

$$\begin{aligned}\frac{d}{dt}\widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_b} &= -\alpha_{i_1 i_2 \ldots i_b}\widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_b} + \zeta_{i_1 i_2 \ldots i_{b-1}, i_1 i_2 \ldots i_b}(\widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_{b-1}} - \widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_b}) \\ &+ \widetilde{I}^{ch}_{i_1 i_2 \ldots i_b}(\widetilde{Y}^{\gamma}_{i_1 i_2 \ldots i_b}, \widehat{R}^{\gamma}) \qquad b = b_1, \ldots, b_K\end{aligned} \tag{6}$$

To be reviewed here: xxxxxFinally, the sum $N(V^{\gamma}, \widetilde{Y}^{\gamma})$ of all longitudinal currents coming from the proximal part of the dendritic tree to the soma has the following form

$$N(V^{\gamma}, \widetilde{Y}^{\gamma}) = \sum_{i_1=1}^{Q^1} \alpha_{i_1}(V^{\gamma} - \widetilde{Y}^{\gamma}_{i_1}/C_{i_1}) \tag{7}$$

This current contributes to $I^{\gamma}(V^{\gamma}, \widetilde{Y}^{\gamma}, \widehat{R}^{\gamma})$ considered in (1). In (5) (resp. (6)), $\widetilde{I}^{ch}_{i_1 i_2 \ldots i_k}(V^{\gamma}, \widehat{R}^{\gamma})$ (resp. $\widetilde{I}^{ch}_{i_1 i_2 \ldots i_b}(V^{\gamma}, \widehat{R}^{\gamma})$) is the ionic channels active transfert current accross compartment $i_1 i_2 \ldots i_k$ (resp. $i_1 i_2 \ldots i_b$). In (7), C_{i_1} is the capacitance of compartment $i_1 = 1 \ldots Q^1$. The right side of (5), (6) constitute the building block of the function $\widetilde{\Xi}^{\gamma}(V^{\gamma}_i, \widetilde{Y}^{\gamma}_i, \widehat{R}^{\gamma}_i)$ which has been introduced in (2).

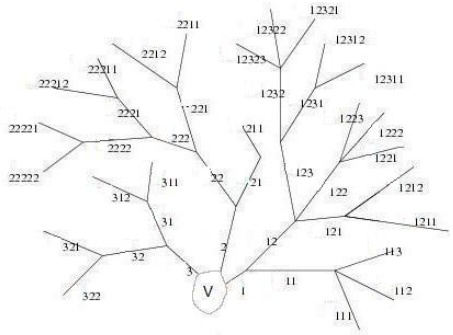


Figure 1 – A typical multicompartments structure

2.3 The inhomogeneous time dependant synaptic interactions model in the system of connected networks $\mathscr{P}_{\gamma}, \gamma = 1, 2..P$.

For the j^{th} cell in $\mathscr{P}_{\gamma}, \gamma = 1, 2..P$, we introduce 3 types of synaptic variables. Some of these variables, which are called $S^{s,\gamma}_j$ and $S^{d,\gamma}_j$, where s and d refer to the soma and dendritic structure of the cell, are useful for the description of the various neurotransmitter mechanisms occuring in the contacts. In the following, we deal with well known neurotransmitters present in cortical neurons and the basal ganglia structure, like glutamate (Glu),γ-aminobutyric acid ($GABA$), acetylcholine (ACh) and dopamine (DA). We introduce also another type of synaptic variable which is called Φ^{γ}_j, for the j^{th} cell in $\mathscr{P}_{\gamma}$. These variables are introduced in order to have some control on the dynamical change of the maximum amplitudes of the synaptic conductances which may occur in cells which will be considered later, like the spiny neurons in the striatum nucleus. The general form

for the synaptic currents occuring in the soma body and dendritic compartmental arborescence of the i^{th} cell in $\mathscr{P}_\gamma$, $I^{s,\gamma}_{i,syn}(t)$ and $\widetilde{I}^{d,\gamma}_{i,syn}(t)$, is the following

$$I^{s,\gamma}_{i,syn}(t) = \sum_{\alpha=1}^{P} \frac{1}{K_\alpha}(W^{s,(\gamma,\alpha)} - V^\gamma_i) \sum_{j=1}^{K_\alpha} \Gamma^{s,(\gamma,\alpha)}(\Phi^\gamma_i, \Phi^\alpha_j) S^{s,\alpha}_j \tag{8}$$

$$\widetilde{I}^{d,\gamma}_{i,syn}(t) = \sum_{\alpha=1}^{P} \frac{1}{K_\alpha}(\widetilde{W}^{d,(\gamma,\alpha)} - \widetilde{Y}^\gamma_i) \sum_{j=1}^{K_\alpha} \Gamma^{d,(\gamma,\alpha)}(\Phi^\gamma_i, \Phi^\alpha_j) S^{d,\alpha}_j \tag{9}$$

In (8) and (9), $W^{s,\gamma}, \widetilde{W}^{d,\gamma}$ are given synaptic potential valued parameters, $\Gamma^{s,d,(\gamma,\alpha)}$ which are maximum amplitudes of the synaptic conductances, are functions of synaptic variables Φ^γ_i and Φ^α_j. In order to specify the way in which the synaptic variables $S^{s,\gamma}_j$, $S^{d,\gamma}_j$ and Φ^γ_j evolve, we introduce the sigmoidal function $\sigma^{s,d}_\Theta$ which is defined in terms of the parameters $U^{s,d}_\Theta$ and $\beta^{s,d}$

$$\sigma^{s,d}_\Theta(u) = (1 + e^{-\beta^{s,d}(u - U^{s,d}_\Theta)})^{-1} \tag{10}$$

In (10), the variable u can designate the membrane potential of soma or that of a dendritic compartment. We introduce the following functions $\mathscr{L}^{s,d,\alpha}(V^\alpha_j, \widetilde{Y}^\alpha_j)$ of the soma potential V^α_j, and the dentritic potential $\widetilde{Y}^\alpha_j$ of the j^{th} presynaptic cell of the network $\mathscr{P}_\alpha$. $\{\widetilde{Y}^\alpha_j\}_k$ means the k^{th} dendritic compartment of the cell.

$$\mathscr{L}^{s,d,\alpha}(V^\alpha_j, \widetilde{Y}^\alpha_j) = \chi^{s,d,\alpha}\sigma^{s,d}_\Theta(V^\alpha_j) + \sum_{k=1}^{d_\alpha} \tau^{s,d,\alpha}_k \sigma^{s,d}_\Theta(\{\widetilde{Y}^\alpha_j\}_k) \tag{11}$$
$$\alpha = 1, 2, \ldots, P \quad j = 1, 2, \ldots, K_\alpha$$

In (11), $\chi^{s,\alpha}, \tau^{s,\alpha}_k$ (resp. $\chi^{d,\alpha}, \tau^{d,\alpha}_k$) are real parameters which are affiliated to presynaptic somas (resp. dendritic compartments). These $\mathscr{L}^{s,d,\alpha}$ functions are introduced in order to detect presynaptic events linked to an activity of sufficiently high amplitude compared to the values of the equilibrium membrane potential.

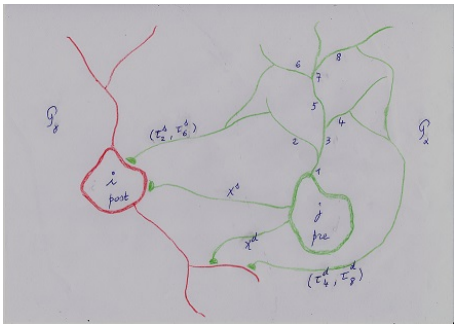


Figure 2 – A rough picture of the connexion model

The dynamical law for the synaptic variables $S^{s,\alpha}_j$ and $S^{d,\alpha}_j$ is postulated of the form

$$\begin{aligned}
\frac{d}{dt}S^{s,\alpha}_j &= \mathscr{L}^{s,\alpha}(V^\alpha_j, \widetilde{Y}^\alpha_j)(1 - S^{s,\alpha}_j) - \kappa^s S^{s,\alpha}_j = \Upsilon^{s,\alpha}(V^\alpha_j, \widetilde{Y}^\alpha_j, S^{s,\alpha}_j) \\
\frac{d}{dt}S^{d,\alpha}_j &= \mathscr{L}^{d,\alpha}(V^\alpha_j, \widetilde{Y}^\alpha_j)(1 - S^{d,\alpha}_j) - \kappa^d S^{d,\alpha}_j = \Upsilon^{d,\alpha}(V^\alpha_j, \widetilde{Y}^\alpha_j, S^{d,\alpha}_j) \\
&\alpha = 1, 2, \ldots, P \quad j = 1, 2, \ldots, K_\alpha
\end{aligned} \tag{12}$$

The coupling coefficients $\Gamma^{s,d,(\gamma,\alpha)}$ in (8), (9) are variable over time and depend on the state of the pre and post synaptic cells according to the following Hebbian form

$$\Gamma^{s,d,(\gamma,\alpha)}(\Phi^\gamma_i, \Phi^\alpha_j) = J^{s,d,(\gamma,\alpha)}\Phi^\gamma_i \Phi^\alpha_j \tag{13}$$

The variation of the synaptic variables Φ^γ_i, $\gamma = 1, .., P$ is modeled in the following way

$$\frac{d\Phi_j^\nu}{dt} = \Omega^\nu(V_j^\nu, \widetilde{Y}_j^\nu, \Phi_j^\nu) \tag{14}$$

In the case where these functions $\Omega^\nu = 0$, the variables Φ_i^γ are in fact constant, therefore the coupling coefficients between cells are constant. These networks have a homogeneous structure. In the case where $\Omega^\nu \neq 0$, the networks are inhomogeneous, the coupling coefficients are variable over time. In summary, the synaptic currents in the formulas (8), (9) have an evolution associated with the states of the membrane potential of the pre and post synaptic cells of the coupled networks, this evolution being also associated with the variations of the other synaptic variables introduced which verify the equations (14).

Actually, all synaptic dynamical laws defined in (23) and (14) may be rewritten in a more concise form, with $\check{Q}_j^\alpha = (S_j^{s,\alpha}, S_j^{d,\alpha}, \Phi_j^\alpha)$ and $\check{\Theta}^\alpha = (\Upsilon^{s,\alpha}, \Upsilon^{d,\alpha}, \Omega^\alpha)$

$$\frac{d}{dt}\check{Q}_j^\alpha = \check{\Theta}^\alpha(V_j^\alpha, \widetilde{Y}_j^\alpha, \check{Q}_j^\alpha) \tag{15}$$

2.4 The mean field approach for coupled networks $\mathscr{P}_\gamma, \gamma = 1, 2..P$.

We have shown ([6]) that it is possible to construct a mean field theory for large-scale neural networks of the type considered in the previous sections, whose monocellular characteristics are described by conductance based systems and whose inhomogeneous synaptic interactions can evolve with time. Actually, in [6], the neuronal model is punctual, with no dendritic structure. We now generalize the framework of this kinetic approach to neural systems with a dendritic tree and more elaborate synaptic connexions.

The variables $Z_k^\gamma = (V_k^\gamma, \widetilde{Y}_k^\gamma, \widehat{R}_k^\gamma, \check{Q}_k^\gamma)$, $Z_k^\gamma \in R^{d_\gamma + m_\gamma + 4}$, $k = 1, 2, \ldots, K_\gamma, \gamma = 1, 2, \ldots, P$, are called state variables of the system.

Let us define $\mathcal{F}^\gamma$, $\mathscr{M}^{\gamma\alpha}$ and ζ^γ by

$$\begin{aligned}
\mathcal{F}^\gamma(Z_i^\gamma) &= ((1/C_s^\gamma)\{I^\gamma(V_i^\gamma, \widetilde{Y}_i^\gamma, \widehat{R}_i^\gamma) + I_{ext}^{s,\gamma}(t)\}, (1/C_d^\gamma)\{\widetilde{\Xi}^\gamma(V_i^\gamma, \widetilde{Y}_i^\gamma, \widehat{R}_i^\gamma) + \widetilde{I}_{ext}^{d,\gamma}(t)\}, \\
&\quad \widehat{\Psi}^\gamma(V_i^\gamma, \widetilde{Y}_i^\gamma, \widehat{R}_i^\gamma),\ \check{\Theta}^\gamma(V_i^\gamma, \widetilde{Y}_i^\gamma, \check{Q}_i^\gamma)) \\
\mathscr{M}^{\gamma\alpha}(Z_i^\gamma, Z_j^\alpha) &= ((1/C_s^\gamma)J^{s,(\gamma,\alpha)}\Phi_i^\gamma\Phi_j^\alpha(W^{s,(\gamma,\alpha)} - V_i^\gamma)S_j^{s,\alpha}, \\
&\quad (1/C_d^\gamma)J^{d,(\gamma,\alpha)}\Phi_i^\gamma\Phi_j^\alpha(\widetilde{W}^{d,(\gamma,\alpha)} - \widetilde{Y}_i^\gamma)S_j^{d,\alpha}, \widehat{0}, \check{0}) \\
\zeta_{i,t}^\gamma &= (\eta_{i,t}^\gamma, \widetilde{0}, \widehat{0}, \check{0})
\end{aligned} \tag{16}$$

$\widetilde{0}$ (resp.$\widehat{0}$) being the null vector in $\mathbb{R}^{d_\gamma}$ (resp.$\mathbb{R}^{m_\gamma}$). The vectors $\mathcal{F}^\gamma$, $\mathscr{M}^{\gamma\alpha}$ and ζ^γ belong to $R^{d_\gamma + m_\gamma + 4}$. With these notations, the dynamical system describing the activity of the neural system constituted by P connected networks $\mathscr{P}_\gamma, \gamma = 1, 2..P$ is the following:

$$\frac{dZ_i^\gamma}{dt} = \mathcal{F}^\gamma(Z_i^\gamma) + \zeta_{i,t}^\gamma + \sum_{\alpha=1}^{P}\frac{1}{K_\alpha}\sum_{j=1}^{K_\alpha}\mathscr{M}^{\gamma\alpha}(Z_i^\gamma, Z_j^\alpha)\ \ i = 1, 2, \ldots, K_\gamma,\ \ \gamma = 1, 2, \ldots, P. \tag{17}$$

In order to elaborate the kinetic theory of the system, we first define a concept of neural probability distribution for each network$\mathscr{P}_\gamma$ and derive a system of coupled non-linear, integropartial differential equations (IPDE) for these distributions. The solutions of these systems of equations called McKean Vlasov Fokker Planck (MVFP) are useful for obtaining statistical measures describing the activity of these networks in the case where they are of large size.

Let us denote $n_t^\gamma(U)$ the neural population probability distributions (PPD) for the networks $\mathscr{P}_\gamma, \gamma = 1, 2..P$, these distributions satisfy the following system of IPDEs (see Appendix 5)

$$\frac{\partial}{\partial t}n_t^\gamma(U) = -\frac{\partial}{\partial U}(\mathcal{F}^\gamma(U)n_t^\gamma(U)) - \frac{\partial}{\partial U}\sum_{\alpha=1}^{P}\int_{R^{d_\alpha + m_\alpha + 4}} dU'\mathscr{M}^{\gamma\alpha}(U, U')n_t^\gamma(U)n_t^\alpha(U') + \frac{1}{2}(\beta_V^\gamma)^2\frac{\partial^2}{\partial v^2}n_t^\gamma(U) \tag{18}$$

$$\gamma = 1, 2, \ldots, P,\ U = (v, \widetilde{y}, \widehat{r}, \check{q}),\ v \in \mathbb{R}, \check{q} = (\mathscr{S}^s, \mathscr{S}^d, \phi) \in \mathbb{R}^3,\ \widetilde{y} \in \mathbb{R}^{d_\gamma},\ \widehat{r} \in \mathbb{R}^{m_\gamma}, \tag{19}$$

$$U' = (v', \widetilde{y}', \widehat{r}', \check{q}'),\ v' \in \mathbb{R}, \check{q}' = (\mathscr{S}^{s\prime}, \mathscr{S}^{d\prime}, \phi') \in \mathbb{R}^3,\ \widetilde{y}' \in \mathbb{R}^{d_\alpha},\ \widehat{r}' \in \mathbb{R}^{m_\alpha}. \tag{20}$$

The variable v (resp. $\widetilde{y}, \widehat{r}, \check{q}$) has soma potential (resp. compartmental dendritic potential, ionic channel gating, synaptic) meaning for cells in $\mathscr{P}_\gamma$. Let us write the equation (18) in terms of the functionals introduced in (16).

$$\begin{aligned}
\frac{\partial}{\partial t} n_t^\gamma(v,\widetilde{y},\widehat{r},\check{q}) = &-(1/C_s^\gamma)\frac{\partial}{\partial v}\{(I^\gamma(v,\widetilde{y},\widehat{r}) + I_{ext}^{s,\gamma}(t))n_t^\gamma(v,\widetilde{y},\widehat{r},\check{q})\} - (1/C_d^\gamma)\frac{\partial}{\partial \widetilde{y}}\{(\widetilde{\Xi}^\gamma(v,\widetilde{y},\widehat{r}) + \widetilde{I}_{ext}^{d,\gamma}(t))n_t^\gamma(v,\widetilde{y},\widehat{r},\check{q})\} \\
&- \frac{\partial}{\partial \widehat{r}}\{\widehat{\Psi}^\gamma(v,\widetilde{y},\widehat{r})n_t^\gamma(v,\widetilde{y},\widehat{r},\check{q})\} - \frac{\partial}{\partial \check{q}}\{\check{\Theta}^\gamma(v,\widetilde{y},\check{q})n_t^\gamma(v,\widetilde{y},\widehat{r},\check{q})\} \\
&- (1/C_s^\gamma)\frac{\partial}{\partial v}\{\phi n_t^\gamma(v,\widetilde{y},\widehat{r},\check{q})\sum_{\alpha=1}^{P} J^{s,(\gamma,\alpha)}(W^{s,(\gamma,\alpha)} - v)E_t^\alpha(\Phi S^s)\} \\
&- (1/C_d^\gamma)\frac{\partial}{\partial \widetilde{y}}\{\phi n_t^\gamma(v,\widetilde{y},\widehat{r},\check{q})\sum_{\alpha=1}^{P} J^{d,(\gamma,\alpha)}(\widetilde{W}^{d,(\gamma,\alpha)} - \widetilde{y})E_t^\alpha(\Phi S^d)\} \\
&+ \frac{1}{2}(\beta_V^\gamma)^2\frac{\partial^2}{\partial v^2} n_t^\gamma(v,\widetilde{y},\widehat{r},\check{q})
\end{aligned} \tag{21}$$

The coupling terms have been written in terms of expectations E_t^α of functionals evaluated in the state space $R^{d_\alpha+m_\alpha+4}$, at time t.

$$E_t^\alpha(\Phi S^{s,d}) = \int_{R^{d_\alpha+m_\alpha+4}} dv'd\widetilde{y}'d\widehat{r}'d\check{q}'\phi'\mathscr{S}^{s,d'}n_t^\alpha(v',\widetilde{y}',\widehat{r}',\check{q}') \tag{22}$$

3 The mean field for the basal ganglia network.

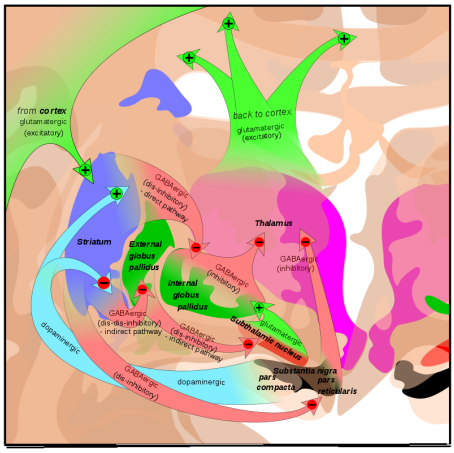


Figure 3 – A rough picture of the basal ganglia network, ref

The dynamical law for the synaptic variables $S_j^{s,\alpha}$ and $S_j^{d,\alpha}$ is postulated of the form

$$\begin{aligned}
\frac{d}{dt}S_j^{s,\alpha} &= \mathscr{L}^{s,\alpha}(V_j^\alpha,\widetilde{Y}_j^\alpha)(1-S_j^{s,\alpha}) - \kappa^s S_j^{s,\alpha} = \Upsilon^{s,\alpha}(V_j^\alpha,\widetilde{Y}_j^\alpha,S_j^{s,\alpha}) \\
\frac{d}{dt}S_j^{d,\alpha} &= \mathscr{L}^{d,\alpha}(V_j^\alpha,\widetilde{Y}_j^\alpha)(1-S_j^{d,\alpha}) - \kappa^d S_j^{d,\alpha} = \Upsilon^{d,\alpha}(V_j^\alpha,\widetilde{Y}_j^\alpha,S_j^{d,\alpha}) \\
&\alpha = 1,2,\ldots,P \quad j = 1,2,\ldots,K_\alpha
\end{aligned} \tag{23}$$

In the following, the general system of MVFP equations for large-scale interacting networks of biologicaly neurons, which has been presented in section 2.4, is applied for a model of basal ganglia XXXXXXXX. Although the system is in principle dedicated to dendritic architectures of any dimension (this in the context of the multicompartmental approximation), this organization will be reduced for certain cells of the networks that we will consider, to a very simple description consisting of a single dendritic compartment associated with a soma and its axon, for other cells, the dendritic structure will only consist of a very limited number of compartments whereas for other cells, the will consist only of a soma and its axon. On the other hand, another simplifying hypothesis will be made at the level of the dynamics of neuromediator transfers. A general model is proposed for these dynamics, the parameters which characterize them which vary according to the mediators considered. The basal ganglia organization model that we are considering also calls for simplifying hypotheses. The proposed

scheme is however considered useful (refs) in the understanding of many mechanisms putting the nuclei of this system into action. This later includes:

- $\mathscr{P}_1$, the subset of striatum (Str) cells which express the $D1$ dopamine receptor
- $\mathscr{P}_2$, the subset of striatum (Str) cells which express the $D2$ dopamine receptor
- $\mathscr{P}_3$, the internal globus pallidus (GPi)
- $\mathscr{P}_4$, the external globus pallidus (GPe)
- $\mathscr{P}_5$, the subthalamic nucleus (STN)
- $\mathscr{P}_6$, the substantia nigra pars compacta (SNc)
- $\mathscr{P}_7$, fast spiking (FS) interneurons connected to $D1$ cells
- $\mathscr{P}_8$, fast spiking (FS) interneurons connected to $D2$ cells

The general current goes from the cortex to the basal ganglia, where the striatum constitutes the entry path while the GPi is the main exit points to the ipsilateral motor thalamic nuclei VA and VL, which returns to the premotor cortex (and additional motor area) responsible for programming the movement.

The wiring among these nuclei is usually described in the following terms [32] ,see also Fig. 4.

a) The striatopallidal pathway is a GABAergic, inhibitory connection between the striatum and both segments of the globus pallidus, GPe and GPi. The medium spiny neurons (MSN) make up a large part of the striatum. These MSNs have radially projecting dendrites that are densely studded with dendritic spines ([8]). These projection neurons of the striatum have two different peptide transmitters being co-localized to define two sets of MSNs, which are called above $\mathscr{P}_2$ and $\mathscr{P}_3$. The first of these groups of MSNs project directly to the GPi while the other group of MSNs projects to the GPe.
b)The nigrostriatal pathway from SNc makes a dopaminergic synapse onto striatal neurons. This is a mixed pathway, with excitatory effects on $D1$ striatal neurons and inhibitory effects on $D2$ striatal cells.
c)The GPe segment makes a GABAergic, inhibitory connection to the STN.
d) The STN makes glutamatergic excitatory connections onto the GPi.
e) The FS interneurons XXXXXXX

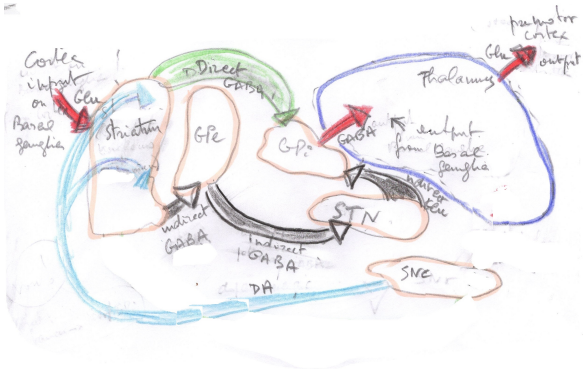
In what follows, we will focus more specifically on the role played by this system in processing cortical information of a general nature and its structure at the output of the GPi. We will not consider the processing of this final signal by the cortico thalamic unit.

3.1 Two pathways process signals in the basal ganglia

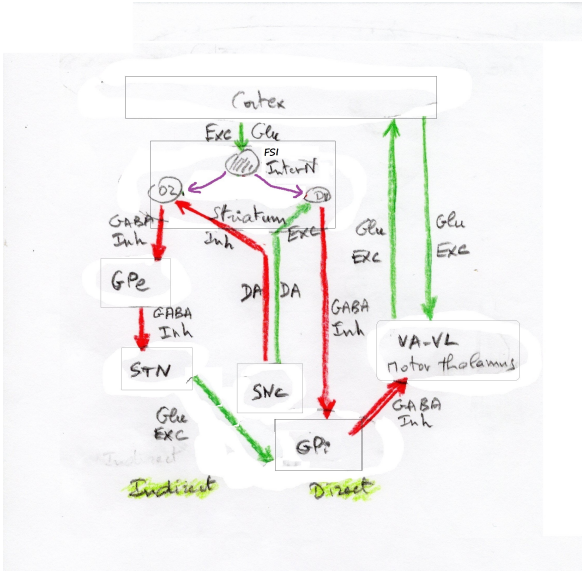
There are two important pathways through which striatal information, coming from the cortex, reaches GPi - the direct pathway and the indirect pathway.These two pathways have opposite effects on motor activity and help explain many clinical symptoms of basal ganglia diseases.

- Direct pathway.
  In the direct pathway, striatal cells project directly to GPi. Using this pathway results in an increase in the excitatory drive from VA and VL to cortex. Actually, cells in GPi have inhibitory action on thalamic nuclei, through GABA transmitters. Striatal cells, which are activated through glutamergic transmitters from cortex, also have an inhibitory action, in this pathway, on the GPi, through GABA trnsmitters. This results in a decrease in the inhibitory activity of GPi on VA/VL, which acts as an excitatory (or dis-inhibition) effect. Finally, activation of this pathway leads to increased firing of the cells of VA/VL and therefore of those of the motor cortex, see Fig. 5.

- Indirect pathway.
  In the indirect pathway, cortical fibers excite striatal neurons that project to GPe. The increased activity of the GABAergic striatal neurons decreases activity in GPe. The GABAergic cells in Gpe inhibit cells in STN, so the decrease in activity in GPe results in less inhibition of cells in STN. That is, subthalmic neurons are dis-inhibited and increase their activity. The projection from STN to GPi is excitatory, so the increased activity in the subthalamic nucleus results in more excitation to cells in GPi. Thus, the end result of actions

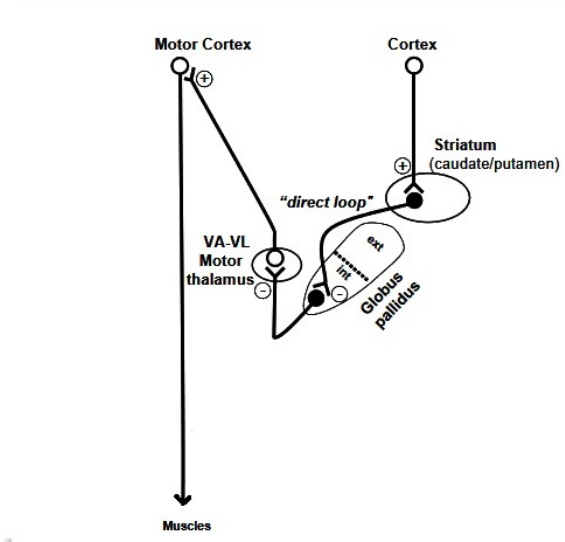


(a) Schematic representation of the direct and indirect pathways in the basal ganglia circuit and dopamine projections. Neurotransmitters : Glu (Glutamate), DA (Dopamine), GABA (Gamma-AminoButyric Acid).

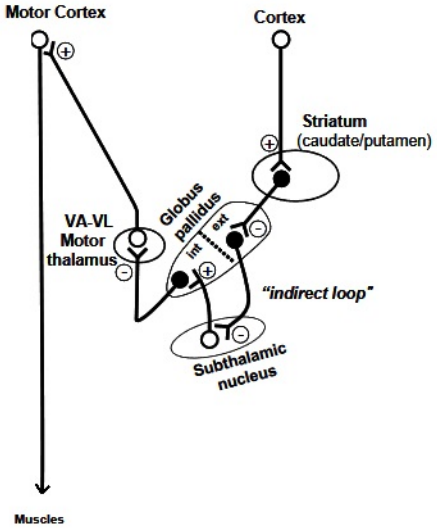


(b) Simplified organization of the basal ganglia connectivity. Exc (resp. Inh): excitatory (resp. inhibitory) connexions. Regulation from SNc on D1 and D2 dopaminergic receptors in striatum.

Figure 4 – XXXX.



(a) Direct pathway.



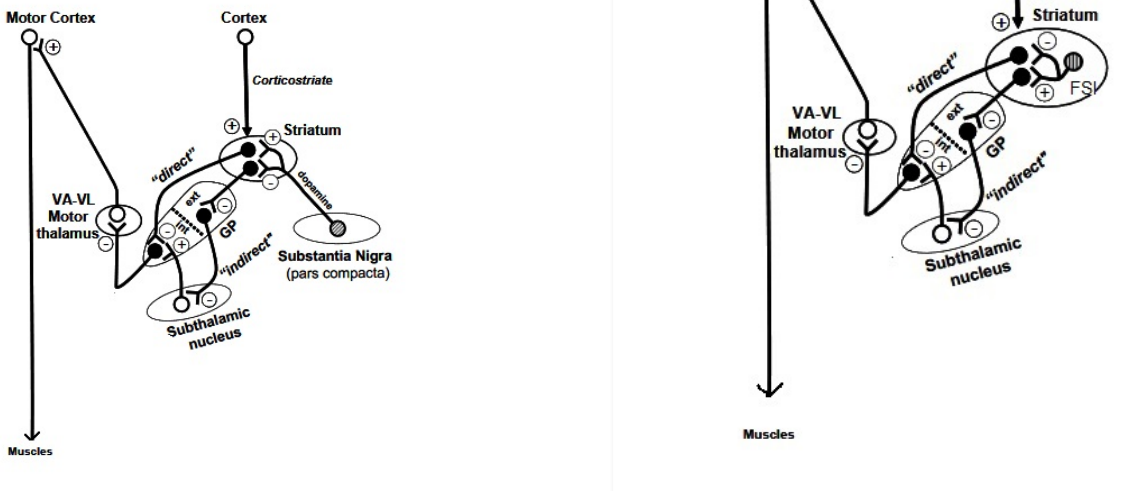
(b) Indirect pathway.

Figure 5 – XXXX.

of the indirect loop is an increase in activity of the GABAergic cells in GPi that project to VA/VL namely an increase in inhibition of the thalamic neurons.The indirect pathway turns down the motor thalamus and, in turn, motor cortex. Thus, it turns down motor activity, see Fig. 5.

- Dopaminergic modulation of direct and indirect pathways.
  Striatal neurons are modulated by two important dopaminergic neuro modulatory systems. Each of these systems differentially affects the direct and indirect pathways, thereby altering their balance and the amount of motor activity that is produced. Dopamine is produced by cells in the pars compacta of the substantia nigra pars compacta (SNc). Nigrostriatal axon terminals release dopamine into the striatum. Dopamine

has an excitatory effect upon cells in the striatum that are part of the direct pathway, via $D1$ receptors. Dopamine has an inhibitory effect upon striatal cells associated with the indirect pathway, via $D2$ receptors. Thus, the direct pathway (which turns up motor activity) is excited by dopamine while the indirect pathway (which turns down motor activity) is inhibited. Both of these effects lead to increased motor activity, see Fig. 6.



(a) Dopaminergic paths.

(b) Fast spiking interneurons (FSI) paths.

Figure 6 – XXXX.

- Action of FS interneurons on direct and indirect pathways.

  XXXXXXX

3.2 The dynamical systems for the various basal ganglia subnets.

In this section, we describe the various ingredients introduced in general equations (16) allowing to analyze the dynamics of the networks of the basal ganglia made up of the networks $\mathscr{P}_\gamma, \gamma = 1, 2..8$. In a first step, the dynamical models of the individual cells in each $\mathscr{P}_\gamma$ are presented.

3.2.1 The model of $D1$ and $D2$ medium spiny neurons (MSN) of the striatum ([2], [4]) (networks $\mathscr{P}_1$ and $\mathscr{P}_2$).

The networks $\mathscr{P}_1$ and $\mathscr{P}_2$ consist of medium spiny neurons (MSN) of the striatum which represent the major part of this nucleus in the subcortical basal ganglia of the forebrain. The essential feature of these cells is the alternating variation of membrane potential between a rest level ($down$) state and a more depolarized level (up) state ([3]). The $MSNs$ being GABAergic, they inhibit their targets of $SNpc$ ($\mathscr{P}_1$). As inputs level on $MSNs$, one has datas from the cerebral cortex and informations by dopaminergic pathways from the $SNpc$. Indeed, $MSNs$ have dopamine receptors. The latter inhibits the $MSNs$ (type $D2$) in the indirect way and excites the $MSNs$ (type D1) in the direct way.

- Model for soma potential dynamics. The temporal evolution of MSN cells is described by a cellular model of the same type as that developed by Gruber, Solla, Surmeier and Houk ([2]). We have added to this model a dendritic structure which is treated according to a multi-compartmental approach (see section 2.2). Three dendritic compartments are considered. A proximal, connected to the soma, for which the membrane potential is described by the variable y_1. Two other compartments are connected in parallel to the proximal,

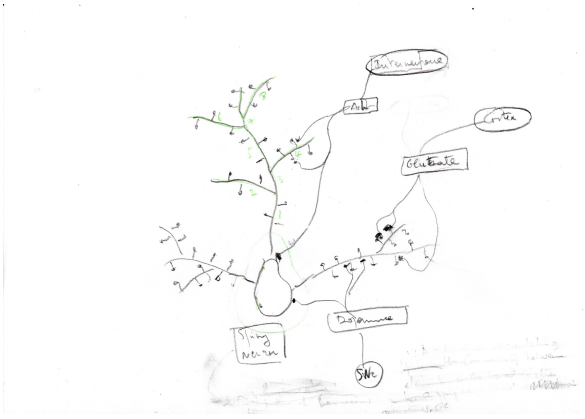


Figure 7 – A schematic representation of the connectivity system of a spiny neuron in striatum.

for which the potential variables are denoted y_{11} and y_{12}. Using the notations introduced in section 2.1, $\widetilde{y} = (y_1, y_{11}, y_{12})$.

As in ([4]), the membrane of a MSN soma is modeled with Hodgkin Huxley type dynamics for active ionic currents. Goldman-Hodgkin-Katz type dynamics (GHK) are used for calcium currents.They are called I_{L-Ca}, are linearly dependent on the concentration of intra- and extracellular calcium $[Ca]_{in}$ and $[Ca]_{ext}$, and not linearly dependent on the membrane potential.Two types of ionic currents are considered, I_{Kir} and I_{Ksi} for which the conductances are potential dependent. It is also introduced a leakage current I_L with a constant conductance. Finally, in order to describe the spiking activity, for variations of the membrane potential of $MSNs$ in the up state, it is proposed here the introduction of Sodium and Potassium currents I_{Na} and I_K which are treated in the classical Hodgkin Huxley formalism.
For this model, the number of variables describing the voltage-gated ions channels activity is $m_2 = 3$.

These variables are the action-inactivation variables which are denoted m, n, h and which are useful for the description of I_{Na} and I_K currents. We call $\widehat{r} = (m, n, h)$. The sum of ionic, active channels and passive leak, transverse currents across the membrane of the soma of $MSNs$ cells $I^1(v, y_1, \widehat{r})$ has the following form, with $g_{d1}(y_1 - v)$ being the longitudinal current coming from the proximal compartment

$$I^k(v, y_1, \widehat{r}) = -(I_{Kir}(v) + I_{Ksi}(v) + I_{L-Ca}(v) + I_{Na}(v, m, h) + I_K(v, n) + I_L(v)) + g_{d1}(y_1 - v) \tag{24}$$

$$k = 1, 2 \tag{25}$$

with

$$\begin{aligned} I_{Kir}(v) =& \widehat{g}_{Kir}(v - E_{Kir})/(1.0 + exp(-(v - V_{h1})/V_{c1})) \\ I_{Ksi}(v) =& \widehat{g}_{Ksi}(v - E_{Ksi})/(1.0 + exp(-(v - V_{h2})/V_{c2})) \\ I_L(v) =& \widehat{g}_L(v - E_L) \\ I_{L-Ca}(v) =& \gamma(v) P_{L-Ca}(v) \end{aligned} \tag{26}$$

where $\gamma(v) = (z^2F^2v/RT)([Ca]_i - [Ca]_{ext}exp(-zFv/RT))/(1 - exp(-zFv/RT))$.
v is the membrane potential of a MSN cell, z is the valence of the Ca^{++} ions, F is Faraday's constant, T is the temperature, R is the gas constant.$[Ca]_i$ (resp.$[Ca]_{ext}$) is intra (resp.extra) cellular Calcium ions concentrations and the voltage dependent membrane permeability is given by

$$P_{L-Ca}(v) = \widehat{P}_{L-Ca}/(1.0 + exp(-(v - V_{Ca-h})/V_{Ca-c})) \tag{27}$$

In relations (26)-(27), E_{Kir}, E_{Ksi} and E_L are reversal potentials, $\widehat{g}_{Kir}$, $\widehat{g}_{Ksi}$, $\widehat{g}_L$ are maximum conductances, V_{h1},V_{c1}, V_{h2}, V_{c2}, V_{Ca-h}, V_{Ca-c} are constants. $\widehat{P}_{L-Ca}$ is the maximum permeability.
Moreover, the Sodium and Potassium currents, $I_{Na}(v, m, h)$ and $I_K(v, n)$, are such that

$$\begin{aligned} I_{Na}(v, m, h) =& \widehat{g}_{Na}m^3h(v - V_{Na}) \\ I_K(v, n) =& \widehat{g}_K n^4(v - V_K) \end{aligned} \tag{28}$$

while $\widehat{\Psi}^k(v,\widehat{r}), k=1,2$ which controls the voltage-gated channels dynamics, are the 3-components functions which have the following structure

$$\begin{aligned}\Psi_i^k(v,\widehat{r}) &= \alpha_i(v)(1-\chi_i) - \beta_i(v)\chi_i \;\; i=1,..,3 \\ \chi_1 &= m, \chi_2 = n, \chi_3 = h \\ k &= 1,2\end{aligned} \tag{29}$$

see ?? for the definition of $\alpha_i(v), \beta_i(v), \;\; i=1,..,3$.

The MSN cells have the particularity of having two operating states, an *up* state, for which the membrane potential variation is in the neighborhood of $-60mV$, where a spiking activity can take place and a *down* state, for which variations are around $-85mV$.The process of passage from one state to another, under the action of synaptic perturbations, can be analyzed by first considering the single-cell dynamics. For a given constant synaptic conductance g_{syn}, corresponding to an input current $I_{syn} = g_{syn}(v - E_S)$, where E_S is a fixed reversal potential, first of all the stationary values V_{stat} of the system (26)-(27) are determined, to which the input has been added. These values should check the relationship

$$\begin{aligned}\mathscr{I}(V_{stat}) =& I_{Kir}(V_{stat}) + I_{Ksi}(V_{stat}) + I_{L-Ca}(V_{stat}) + \widehat{g}_{Na} m_\infty(V_{stat})^3 h_\infty(V_{stat})(V_{stat} - V_{Na}) \qquad (30)\\ &+ \widehat{g}_K n_\infty(V_{stat})^4 (V_{stat} - V_K) + I_L(V_{stat}) + g_{syn}(V_{stat} - E_S) = 0 \qquad (31)\end{aligned}$$

where $m_\infty, n_\infty, h_\infty$ are asymptotic values of m, n, h. Fig.8̇ shows the behavior of $\mathscr{I}(v)$ for different values of g_{syn}. For $g_{syn} = g_{min}$ and $g_{syn} = g_{max}$, the curves are tangent to the v axis. For $g_{min} < g_{syn} < g_{max}$, the relation (31) has 3 solutions, 2 of which correspond to stable equilibrium points and 1 an unstable equilibrium.The diagram of equilibrium points according to parameter g_{syn} is shown in Fig.8̇. It indicates a bifurcation between states with low equilibrium values (*down* states) and states with higher values (*up* states), when g_{syn} varies.The system is now subject to synaptic inputs $\mathscr{G}_{syn}(t)$ with strong temporal variations in a domain near the $[g_{min}, g_{max}]$ interval. A typical example of synaptic signal is shown in Fig.8̇.
The synaptic input model $\mathscr{G}_{syn}(t)$ is built in the following way. First, a temporal division $\{t_i\}\; i=1,..,N_{inp}$ is built such that $\Delta t_i = t_{i+1} - t_i = \zeta$ where $\zeta = \mathscr{U}(0,\delta)$ is a continuous random variable uniformly distributed on $[0,\delta]$. Let $\Gamma(t)$, a continuous function, whose variations are bounded. Using the temporal division $\{t_i\}\; i=1,..,N_{inp}$, we construct a set of values $\left\{ampl_i = g_{min} + (1/2)(g_{max} - g_{min})(1 + \Gamma(t_i) + \xi)\right\}_{i=1,..,N_{inp}}$ where $\xi = \mathscr{U}(0,\phi)$. $\mathscr{G}_{syn}(t)$ is then obtained by linear interpolations between the values $\{ampl_i\}_{i=1,..,N_{inp}}$. In Fig.8̇, $\Gamma(t) = \Gamma_{max}(sin(\omega_1 t) + cos(\omega_2 t) + sin(\omega_3 t))$ where $\omega_i = 2\pi/T_i, \;\; i=1,..,3$.

The response of an MSN cell to this kind of input is shown in Fig.8̇ where a switchover occurs alternately from an *up* state (possibly producing spikes) to a *down* state as variations of $\mathscr{G}_{syn}(t)$ occur.
See Appendix 4 for the values of the various parameters used in the MSN cells simulations.

- Model for dendritic potential dynamics.

  In the dendritic model of potential activity which is presented below, only the proximal compartment is provided with ion channels having potential dependent active properties. These channels are of the same type as those considered for soma. The associated current is therefore

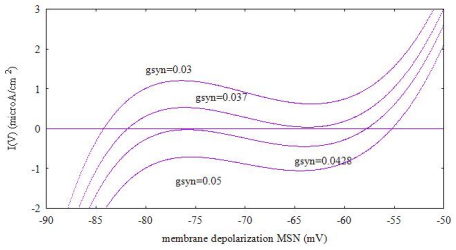
  $$I^{ch}(y_1,\widehat{r}) = -(I_{Kir}(y_1) + I_{Ksi}(y_1) + I_{L-Ca}(y_1) + I_{Na}(y_1,m,h) + I_K(y_1,n))$$

  The set of passive, transverse and longitudinal currents across and along the membrane of the dendritic compartments, as well as the active properties across the proximal compartment, is described by the three dimensional vectors $\widetilde{\Xi}^k(v,\widetilde{y},\widehat{r}), k=1,2$ whose components are.

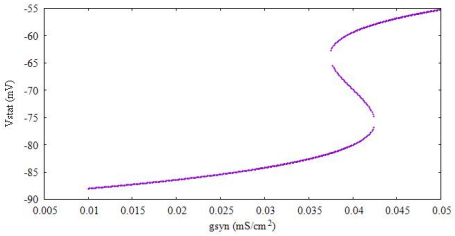
$$\begin{aligned}\Xi_1^k(v,\widetilde{y},\widehat{r}) =& -g_{d1}(y_1 - v) - (g_d + g_{d11} + g_{d12})(y_1 - E_L)\\ &+ g_{d11}(y_{11} - E_L) + g_{d12}(y_{12} - E_L) + I^{ch}(y_1,\widehat{r})\\ \Xi_2^k(v,\widetilde{y},\widehat{r}) =& g_{d11}(y_1 - E_L) - (g_d + g_{d11})(y_{11} - E_L)\\ \Xi_3^k(v,\widetilde{y},\widehat{r}) =& g_{d12}(y_1 - E_L) - (g_d + g_{d12})(y_{12} - E_L)\\ k &= 1,2\end{aligned} \tag{32}$$

  The parameters E_L are equilibium values and $g_d, g_{d1}, g_{d11}, g_{d12}$ have the following meaning, (see 2.2). g_d is the capacitance, assumed identical for each compartment, for the transverse current through these compartments.

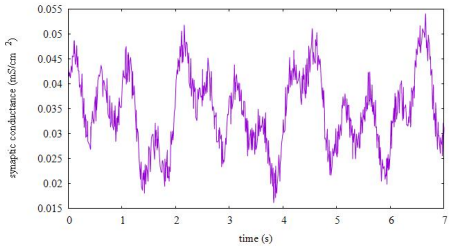
g_{d1} is the conductance for longitudinal current between the proximal compartment and the soma, g_{d11}, g_{d12} are conductances for longitudinal current between each parallel compartment and the proximal one.



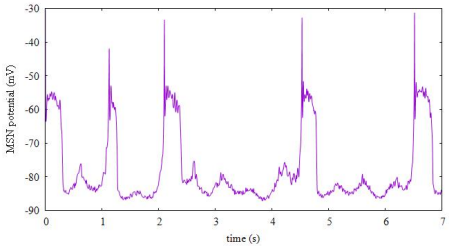
(a) $\mathscr{I}(v)$ curves for different constant synaptic conductances.



(b) Bifurcation diagram.



(c) Synaptic conductance variations.



(d) Membrane potential variations.

Figure 8 – Dynamical behaviour of one isolated MSN cell subjected to corticostriatal synaptic inputs, modified from Gruber et al. ([2]).

3.2.2 The Guo Rubin model ([5]) of the neuronal states in internal, external globus pallidus and subthalamic networks GPi, GPe and STN ($\mathscr{P}_3..\mathscr{P}_5$).

The dynamics of the GPi, GPe and STN cells that make up the $\mathscr{P}_3, \mathscr{P}_4$ and $\mathscr{P}_5$ networks are governed by conductance-based , single -compartment systems originally developed by Guo and Rubin ([5]).

- Internal globus pallidus neurons in GPi ($\mathscr{P}_3$).

  For the model that describes the activity of GPi cells, the variables $h, n, u, [Ca]$ are needed to describe the voltage-gated ion channels including calcium activity. We call $\widehat{r} = ([Ca], h, n, u)$.The sum of ionic, active channels and passive leak, transverse currents across the membrane of the soma $I^3(v, , \widehat{r})$ of the GPi cells have the following forms:

$$I^3(v, \widehat{r}) = -(I_{Na}(v, h) + I_K(v, n) + I_T(v, u) + I_{Ca}(v) + I_{AHP}(v, [Ca]) + I_L(v)) \tag{33}$$

with

$$\begin{aligned} I_{Na}(v, h) =& g_{Na}\, m_\infty(v)^3\, h\, (v - V_{Na}) \\ I_K(v, n) =& g_K\, n^4\, (v - V_K)) \\ I_T(v, u) =& g_T a_\infty(v)^3\, u\, (v - V_{Ca}) \\ I_{Ca}(v) =& g_{Ca} s_\infty(v)^2 (v - V_{Ca}) \\ I_{AHP}(v, [Ca]) =& g_{AHP}\, ([Ca]/([Ca] + k))\, (v - V_K) \\ I_L(v) =& g_L\, (v - V_L) \end{aligned} \tag{34}$$

Moreover, $\widehat{\Psi}^3(v, \widehat{r})$ which controls the voltage-gated channels dynamics, is the 4-components function which

has the following structure

$$\begin{aligned}\Psi_1^3(v,[Ca],u) =&\epsilon\big\{-I_{Ca}(v)-I_T(v,u)-k_{Ca}[Ca]\big\}\\ \Psi_2^3(v,h) =&\Phi_h(h_\infty(v)-h)/\tau_h(v)\\ \Psi_3^3(v,n) =&\Phi_n(n_\infty(v)-n)/\tau_n(v)\\ \Psi_4^3(v,u) =&\Phi_u(u_\infty(v)-u)/\tau_u\end{aligned} \tag{35}$$

with

$$\begin{aligned}X_\infty(v) =&1/(1+e^{-(v-\theta_X)/\sigma_X})\\ X =&h,n,u,m,a,s\\ \tau_X(v) =&\tau_X^0+\tau_X^1/(1+e^{-(v-\theta_X^\tau)/\sigma_X^\tau})\ \ X=h,n\end{aligned} \tag{36}$$

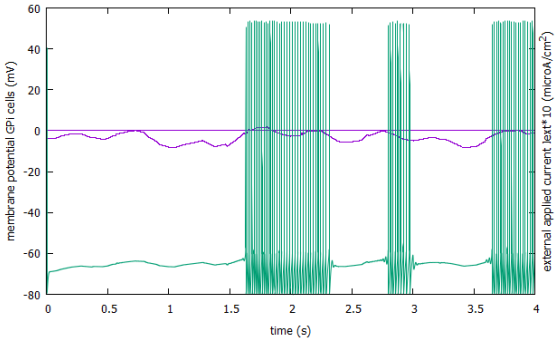


Figure 9 – Bursting activity of isolated GPi cells under external irregular input currents from Guo and Rubin ([5]).

- External globus pallidus neurons in GPe ($\mathscr{P}_4$).

  The activity of GPe cells is treated using the same variables and equations as for GPi cells, with the same parameters. Thus, the sum of ionic, active channels and passive leak, transverse currents across the membrane of the soma $I^4(v,,\widehat{r})$ of the GPe cells have the following forms

$$I^4(v,\widehat{r}) = -(I_{Na}(v,h)+I_K(v,n)+I_T(v,u)+I_{Ca}(v)+I_{AHP}(v,[Ca])+I_L(v)) \tag{37}$$

- Subthalamic nucleus network STN ($\mathscr{P}_5$).

  The dynamics of STN cells is modeled by the same system of equations as that corresponding to the networks GPi and GPe, except for the current I_T and the parameter $\tau_u(v)$) which is a constant for the cells of GPi and GPe in (36) and which is not constant for the cells of the STN. These new parameter functions are

$$\begin{aligned}I_T(v,u) =&g_T a_\infty(v)^3\, b_\infty(u)\,(v-V_{Ca})\\ \tau_u(v) =&\tau_u^0+\tau_u^1/(1+e^{-(v-\theta_u^\tau)/\sigma_u^\tau})\end{aligned} \tag{38}$$

with

$$b_\infty(u) = 1/(1+e^{-(u-\theta_b)/\sigma_b})-1/(1+e^{-\theta_b/\sigma_b}) \tag{39}$$

See Appendix 4 for the values of the various parameters used in the GPi, GPe and STN network simulations.

3.2.3 The Li Bertram Rinzel model ([1]) of dopaminergic bursting cells in the Sbustantia Nigra pars compacta (network $\mathscr{P}_6$).

The network $\mathscr{P}_6$ is built of dopaminergic cells in Sbustantia Nigra pars compacta (SNc) whose dynamical behaviour is governed by a system which is adapted from a model orginally developed by Li, Bertram and Rinzel ([1]). It is made of two compartments, a soma compartment and a dendritic compartment that represents the

distal dendrites, lumped together. So, in this case, $d_6 = 1$. For this model, the number of variables needed to describe the voltage-gated ion channels activity is $m_6 = 6$. These variables include the concentration of Calcium $[Ca]$ in the soma, the concentration of Sodium $[Na]$ in the dendrite, action-inactivation variables denoted h, n, m_T, h_T. We call $\widehat{r} = (h, n, m_T, h_T, [Ca], [Na])$. The sum of ionic, active channels and passive leak, transverse currents across the membrane of the soma $I^6(v, y_1, \widehat{r})$ and across the membrane of the dendrite $\Xi^6(v, y_1, \widehat{r})$, have the following forms

$$\begin{aligned} I^6(v, y_1, \widehat{r}) =& - I_{Na,S}(v, h) - I_{Ca-T}(v, m_T, h_T) \\ & - I_{KDRS}(v, n) - I_{K(Ca)}(v, [Ca]) - g_c/p_{DA}(v - y_1) \\ \Xi^6(v, y_1, \widehat{r}) =& - I_{KDRD}(y_1, n) - I_{NMDA}(y_1) \\ & - I_{pump}([Na]) - I_L(y_1) - g_c/(1 - p_{DA})(y_1 - v) \end{aligned} \tag{40}$$

with

$$\begin{aligned} I_{NaS}(v, h) =& g_{Na}\, m_\infty(v)^3\, h\, (v - V_{Na}) \\ I_{Ca-T}(v, m_T, h_T) =& g_{CaT}\, m_T^2\, h_T\, (v - V_{Ca}) \\ I_{KDRS}(v, n) =& g_{KDRS}\, n^2\, (v - V_K) \\ I_{K(Ca)}(v, [Ca]) =& g_{K(Ca)}\, ([Ca]^4/([Ca]^4 + K_{Ca}^4))\, (v - V_K) \\ m_\infty(v) =& 1/\{(1 + exp(-(v + 35)/6.2)\} \end{aligned} \tag{41}$$

and with

$$\begin{aligned} I_{KDRD}(y_1, n) =& g_{KDRD}\, n^2\, (y_1 - V_K) \\ I_{NMDA}(y_1) =& Cond_{NMDA}(y_1)\, (y_1 - V_{NMDA}) \\ Cond_{NMDA}(y_1) =& g_{NMDA}\{1 + [Mg^{++}]_0/K_{Mg}\, exp(-y_1/q)\}^{-1} \\ I_{pump}([Na]) =& R_{pump}(\Gamma([Na]) - \Gamma([Na], eq)) \\ \Gamma([Na]) =& [Na]^3/([Na]^3 + K_p^3) \\ I_L(y_1) =& g_L(y_1 - V_L) \end{aligned} \tag{42}$$

Moreover, $\widehat{\Psi}^6(v, y_1, \widehat{r})$ which controls the voltage-gated channels dynamics, is the 6-components function which has the following structure

$$\Psi_1^6(v, y_1, \widehat{r}) = \alpha\{ - I_{Na,NMDA}(y_1) - 3I_{pump}([Na])\} \tag{43}$$

$$\Psi_2^6(v, y_1, \widehat{r}) = - \beta I_{Ca-T}(v, m_T, h_T) - k_{Ca}[Ca] \tag{44}$$

$$\begin{aligned} \Psi_i^6(v, y_1, \widehat{r}) =& 1/\tau_i(v)(w_{\infty,i}(v) - w_i) \;\; i = 3, .., 6 \\ & w_3 = h, w_4 = n, w_5 = m_T, w_6 = h_T \end{aligned} \tag{45}$$

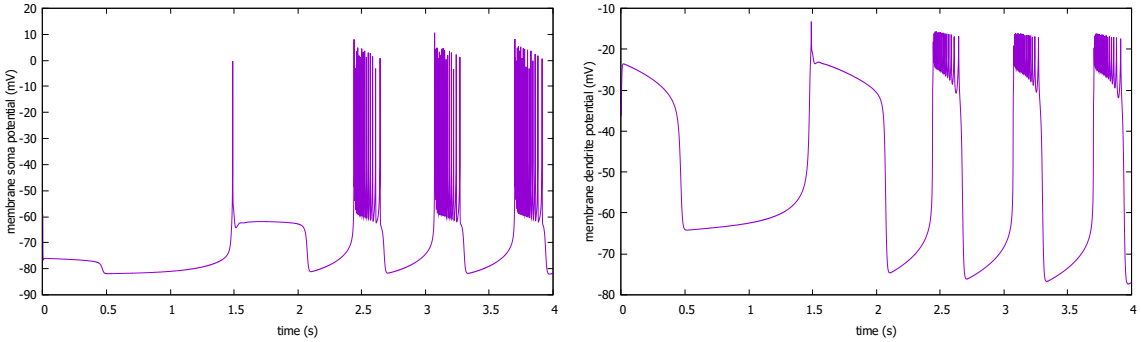
with

$$I_{NaNMDA}(y_1) = g_{NaNMDA}\{1 + [Mg^{++}]_0/K_{Mg}\, exp(-y_1/q)\}^{-1}\, (y_1 - V_{Na}) \tag{46}$$

and

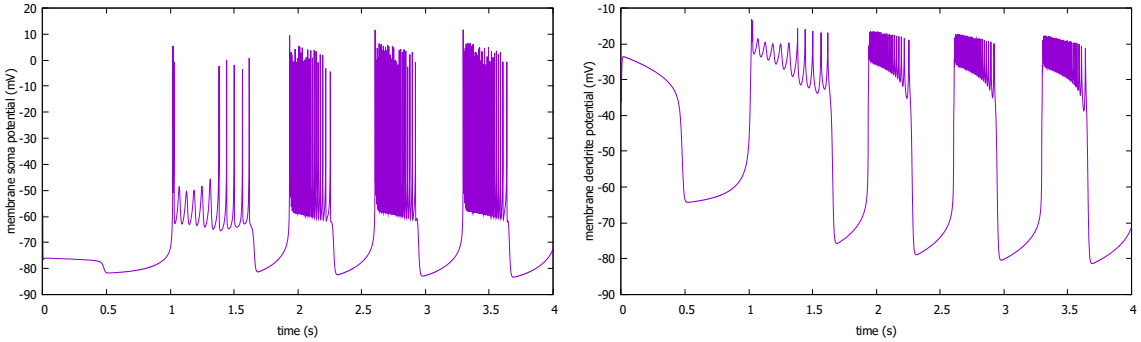
$$\begin{aligned} \nu_3(v) =& \nu_h(v) = \left\{0.4\, \{1 + 2/(1 + exp((v + 25)/5))\}\right\}^{-1} \\ w_{\infty,h}(v) =& 1/\{1 + exp((v + 30)/8.3)\} \\ \nu_4(v) =& \nu_n(v) = (1 + exp(-(v + 70)/10))/\{0.8(1 + 2/(1 + exp((v + 25)/10)))\} \\ w_{\infty,n}(v) =& 1/\{1 + exp(-(v + 31)/5.3)\} \\ \nu_5(v) =& \nu_{m_T}(v) = 1 \\ w_{\infty,m_T}(v) =& 1/\{1 + exp(-(v + 55)/7)\} \\ \nu_6(v) =& \nu_{h_T}(v) = 0.1 \\ w_{\infty,h_T}(v) =& 1/\{1 + exp((v + 81)/11)\} \end{aligned} \tag{47}$$

See Appendix 4 for the values of the various parameters used in the DA cells simulations in Fig. 11.
XXXXXXXXComments on figures here.



(a) Somatic membrane potential. (b) Dendritic membrane potential.

Figure 10 – Bursting activity of isalated autonomous DA cells, $k_{Ca} = 1.0$.



(a) Somatic membrane potential. (b) Dendritic membrane potential.

Figure 11 – Bursting activity of isalated autonomous DA cells, $k_{Ca} = 1.5$, from Li et al. ([1]).

3.2.4 The Golomb, Donner, Shacham, Shlosberg, Amitai, Hansel model ([36]) of striatal fast spiking (FS) interneurons (networks $\mathscr{P}_7$ and $\mathscr{P}_8$).

The $\mathscr{P}_7$ and $\mathscr{P}_8$ networks of FS interneurons in the striatum are made up of cells whose evolution of the membrane potential was modeled by Golomb and his collaborators. This model is made of a single somatic compartment while the number of variables describing the activity of the channels is $m_7 = m_8 = 4$. The equation of the current balance is

$$\begin{aligned} I^k(v,\widehat{r}) = &- I_{Na}(v,h) - I_{Kdr}(v,n) - I_d(v,a,b) - g_L(v - V_L) \\ & k = 7,8 \end{aligned} \tag{48}$$

where v is the membrane potential of the soma, I_{Na} is the Na^+ current for which the gating variable m is instantaneous, reaching the value dependent potential limit m_∞ and h evolves according to a Hodgkin Huxley type dynamic (HH) potential dependent. On the other hand, I_{Kdr} is a delayed rectifier Potassium current, the HH gating variable being n. Finally, a potassium current I_d, which has a fast activation and a slow inactivation, is expressed in terms of the gating variableq a and b (70) .. In addition, using our notations (see []), the Psik functions .....

$$\begin{aligned} I_{Na}(v,h) &= g_{Na} m_\infty^3(v) h (v - V_{Na}) \\ I_{Kdr}(v,n) &= g_{Kdr} n^2 (v - V_K) \\ I_d(v,a,b) &= g_d a^3 b (v - V_K) \end{aligned} \tag{49}$$

with

$$m_\infty(v) = \{1 + exp(-(v - \theta_m)/\sigma_m)\}^{-1} \tag{50}$$

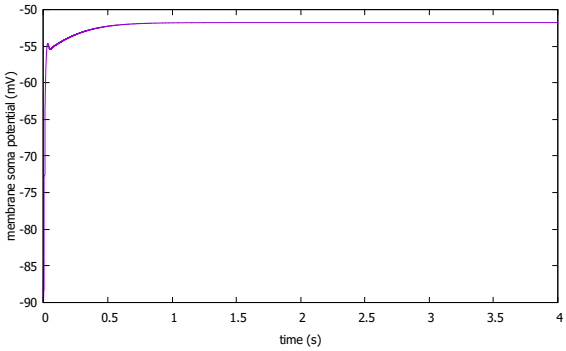
In addition, (see (3)), the 4-components functions $\widehat{\Psi}^k(v,\widehat{r}), k=7,8$, which control the voltage-gated channels dynamics, have the following form

$$\begin{aligned}\Psi_i^k(v,\widehat{r}) &= (w_{i,\infty}(v)-w_i)/\tau_i(v) \quad i=1,..,4 \\ w_1 &= h, w_2 = n, w_3 = a, w_4 = b, \quad k=7,8\end{aligned} \tag{51}$$

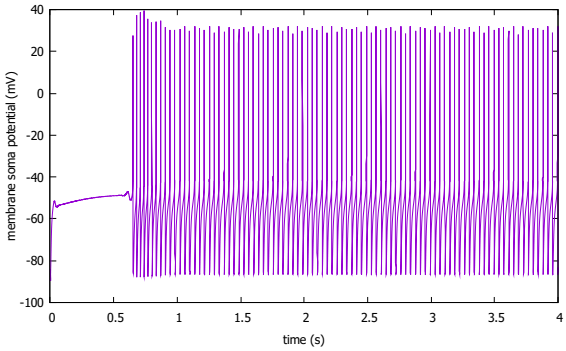
with

$$\begin{aligned}(w_{i,\infty}(v) &= \{1+exp(-(v-\theta_{w_i})/\sigma_{w_i})\}^{-1} \quad i=1,..,4 \\ w_1 &= h, w_2 = n, w_3 = a, w_4 = b \\ \tau_h(v) &= 0.5+14\{1+exp(-(v-\theta_{th})/\sigma_{th})\}^{-1} \\ \tau_n(v) &= \Big\{0.087+11.4/\big\{1+exp((v+14.6)/8.6)\big\}\Big\}\Big\{0.087+11.4/\big\{1+exp(-(v-1.3)/18.7)\big\}\Big\} \\ \tau_a(v) &= \tau_a, \tau_b(v) = \tau_b\end{aligned} \tag{52}$$

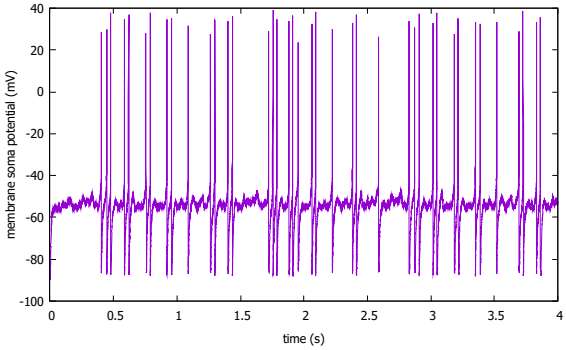
See Appendix 4 for the values of the various parameters used in the FS interneurons simulations.



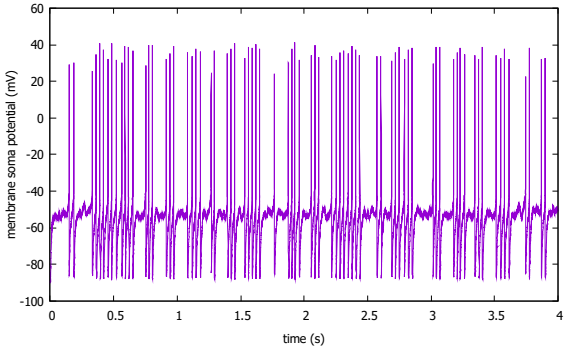
(a) No noise, a step current ($I_{Ext}(t) = 3.0\mu A/cm^2$) is applied. Depolarization of the membrane potential but no production of action potentials.



(b) No noise, a step current ($I_{Ext}(t) = 3.2\mu A/cm^2$) is applied. Emergence of a delayed tonic firing.



(c) A step current ($I_{Ext}(t) = 3.0\mu A/cm^2$) is applied. A noise of amplitude XX causes the appearance of an irregular activity in bursts of spikes.



(d) For $I_{Ext}(t) = 3.2\mu A/cm^2$ and a noise of the same amplitude, the irregular bursting activity is more intense.

Figure 12 – Noise induced activity of FSI cells from Golomb et al.([36]).

3.3 The connectivity model of the basal ganglia.

In this section, wel introduce the ingredients useful for the characterization of the various synaptic currents between cells of the same population and between cells of different populations. The sigmîdal functions (10) and the κ^s parameters in (23) were chosen identical for all the contacts.

3.3.1 Interactions in Str between $D1MSN$ cells in $\mathscr{P}_1$ and $D2MSN$ cells in $\mathscr{P}_2$.

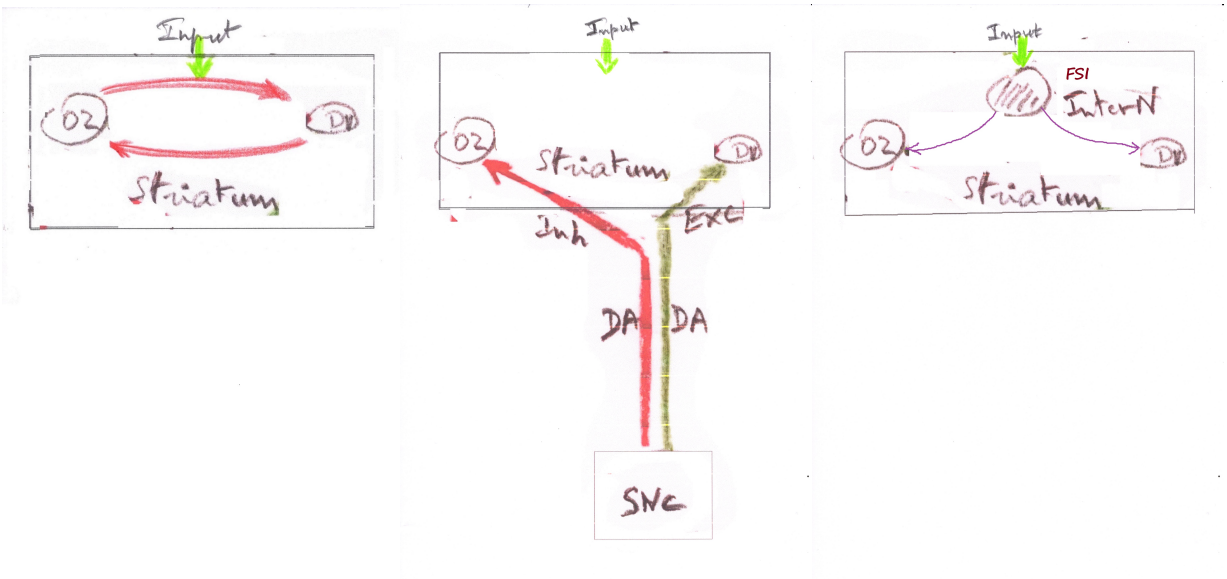
The MSN cell model of the striatum that we are considering consists of a soma and a 3-compartment dendritic system. Referring to the general expression of synaptic currents (8) (9), the following parameters are introduced in order to describe the interactions between $D1MSN$ cells of $\mathscr{P}_1$, between $D2MSN$ cells of $\mathscr{P}_2$, and between $\mathscr{P}_1$ cells and $\mathscr{P}_2$ cells (Fig. 13) : $W^{s(1,1)}, W^{s(2,2)}, W^{s(1,2)}, W^{s(2,1)}$ are "equilibrium" XX values used to specify the excitatory or inhibitory nature of the connection, synaptic contacts taking place on the soma of postsynaptic cells. $\widetilde{W}^{d,(1,1)}, \widetilde{W}^{d,(2,2)}, \widetilde{W}^{d,(1,2)}, \widetilde{W}^{d,(2,1)}$ are vectors with 3 components of the" equilibrium" XX values (for synaptic contacts taking place on the dendrites of postsynaptic cells).
$D1MSN$ and $D2MSN$ cells are $GABA$ergic. Equations (23) model the transfers of $GABA$ neuromediators. The $\mathscr{L}^{s,d}$ functions (11), which enter into the dynamics of the synaptic variables $\mathscr{S}_j^{s,\gamma}$ and $\mathscr{S}_j^{d,\gamma}$ (for the j^{th} cell, $\gamma = 1, 2$), through these equations, are defined using the following parameters : $\chi^{s,1}, \chi^{s,2}, \chi^{d,1}, \chi^{d,2}$, $\tau_k^{s,1}, \tau_k^{s,2}, \tau_k^{d,1}, \tau_k^{d,2}$ $k = 1, 2, 3$. These are real parameters which are affiliated to somas and dendritic compartments. For these later, they are taken identical.
Finally, the connection system of MSN cells in the striatum is completed by a Hebbian approach to the maximum values of synaptic conductances $\Gamma^{s,(\gamma,\alpha)}$, $\Gamma^{d,(\gamma,\alpha)}$ $\gamma = 1, 2$, which are time dependent. The latter have the form

$$\Gamma^{s,(\gamma,\alpha)}(\Phi_i^\gamma, \Phi_j^\alpha) = J^{s,(\gamma,\alpha)} \Phi_i^\gamma \Phi_j^\alpha \quad i = 1, 2, \ldots, K_\gamma, \quad j = 1, 2, \ldots, K_\alpha \quad \alpha, \gamma = 1, 2 \tag{53}$$

The law of evolution of variables Φ_i^γ which was formulated in general in (14), takes the following form, for the i^{th} MSN cell in $\mathscr{P}_1$ and the j^{th} MSN cell in $\mathscr{P}_2$

$$\Omega^1(V_i^1, \Phi_i^1) = -\alpha^1 \Phi_i^1 + \beta^1 \sigma^s(V_i^1)$$
$$\Omega^2(V_j^2, \Phi_j^2) = -\alpha^2 \Phi_j^2 + \beta^2 \sigma^s(V_j^2) \tag{54}$$
$$\tag{55}$$

where σ^s is a sigmoïdal function of the same type as in 10, $\alpha^\gamma, \beta^\gamma, \gamma = 1, 2$ being real parameters. For all the other cells in the Basal Ganglia system, the Φ_j^γ variables, are in fact considered to be constant over time, for $\gamma = 3, .., 7$ and each j^{th} cell.



(a) Coupling between $D1MSN$ and $D2MSN$ cells in striatum under cortical input.

(b) Interactions between SNc and STr.

(c) Interneurons model.

Figure 13 – XXXX.

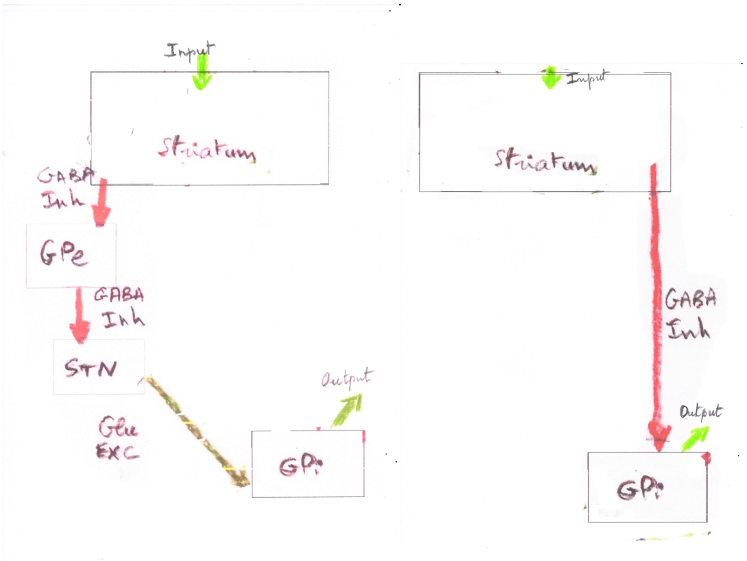
3.3.2 Interactions between $D1MSN$ cells ($\mathscr{P}_1$) and $D2MSN$ cells ($\mathscr{P}_2$) with SNc cells ($\mathscr{P}_6$) and interneurons ($\mathscr{P}_7$ and $\mathscr{P}_8$).

The connection model between SNc cells (resp. interneurons) and $D1MSN$ (resp. $D2MSN$) cells is a more simple model than that considered in the previous section insofar as the connections between cells are made at the somas level (Fig. 13). Connections between SNc cell (resp. interneurons) somas and dendritic compartments of $D1MSN$ (resp. $D2MSN$) cells are not considered. The parameters which control the dopaminergic (resp. cholinergic) character of the transmission originating from SNc cells (resp. interneurons) and projecting onto $D1MSN$ (resp. $D2MSN$) cells are noted, following (8), (11): for the equilibrium values, achieving the excitatory or inhibitory character of the connection, $W^{s,(1,6)}$ (resp. $W^{s,(2,6)}$) for SNc connections, $W^{s,(1,7)}$ (resp. $W^{s,(2,7)}$) and $W^{s,(1,8)}$ (resp. $W^{s,(2,8)}$) for interneurons connections. The parameters of the $\mathscr{L}^{s,d}$ functions for SNc connections are $\chi^{s,6}$. They are $\chi^{s,7}$ and $\chi^{s,8}$ for interneurons connections. These later parameters are identical, in each case, for the $D1MSN$ and $D2MSN$ cells.

3.3.3 Interactions between $D1MSN, D2MSN$ cells in Str and cells of GPi ($\mathscr{P}_3$).

Firstly, let us consider the direct pathway between Str and GPi. As in the previous section, the connectivity model between $D1MSN$ and GPi cells comes down to contacts taking place at the somas of the 2 types of cells (Fig. 14).The parameters used to provide $D1MSN$ cells with GABAergic properties are $W^{s,(3,1)}$ and $\chi^{s,1}$, this later is the parameter for the corresponding $\mathscr{L}^{s,d}$ function.

Along the indirect pathway, the transmission between $D2MSN$ cells and those of GPe ($\mathscr{P}_4$) is also done in a GABAergic manner. Here, the parameters are $W^{s,(4,2)}$ and $\chi^{s,2}$. Between GPe and STN ($\mathscr{P}_5$), the neuromediators are also GABA. The parameters are $W^{s,(5,4)}$ and $\chi^{s,4}$. Finally, between STN ($\mathscr{P}_5$) and GPi ($\mathscr{P}_3$), the connection is Glutamergic. The settings here are $W^{s,(3,5)}$ and $\chi^{s,5}$ (see (Fig. 14)).
The various characteristics of the neural subnetworks of the simplified system of the Basal Ganglia having been presented, we give in what follows an application of the general method (section (??)) allowing to obtain informations, within the framework of approximations due to noises of small amplitude called $DSAM$, on the evolution of statistical quantities such as mean values, variances, covariances of state variables such as membrane potential of somas, dendritic compartments or indeed activation and inactivation variables of ionic channels of the various types of cells considered in the constitution of the system. Recall that the $DSAM$ method is based on a set approach of the mean field type where the various subsystems are made up of a very large number of cells. Finally, the procedure followed will be that of finding relations, along the rules of $DSAM$, between cortical entries into the striatum and outputs taking place at the GPi level.



(a) Indirect pathway. (b) Direct pathway.

Figure 14 – XXXX.

4 Numerics and parameter settings.

Numerical simulations were performed using double precision floating point arithmetic on a Intel Xeon computer with 32 processors. Euler's method was used to solve the system of stochastic differential equations (1)-(3), with a time step $\delta t = 0.01s$. The ordinary differential equations of the DSAM system (approximate moments) were solved using a 4^{th} order RungeKutta method, with the same time step. The number of cells for the $\mathscr{P}_1$ and $\mathscr{P}_2$ networks has been set at $K_1 = K_2 = 100$ while the number of trials is $\mathscr{N}_{trials} = 50$.

The means and variances were evaluated according to the same scheme as described in [6]. In the evaluation of the various moments calculated from the DSAM system, the difficulty lies essentially in the appearance of divergences occurring after a time that generally corresponds to a relatively limited number of patterns. However, these discrepancies can be controlled by a detailed analysis of the initial conditions. When this control is obtained, the comparison of the computation times required to solve DSAM and stochastic systems is clearly to the advantage of the first method.

For example, for 2 coupled systems such as $\mathscr{P}_1$ and $\mathscr{P}_2$, consisting of $K_1 = K_2 = 100$ cells, with a number of dendritic compartments of small size $(d_1 = 1, d_2 = 3)$ and ionic variables in relatively small number $(m_1 = 6, m_2 = 2)$, the number of random equations is $(d_1 + m_1 + 1) * K_1 + (d_2 + m_2 + 1) * K_2$, to which are added $(K_1 + K_2)$ equations for the connection variables. Finally, the number of equations to be solved is $\sum_{\gamma=1}^{2}(d_\gamma + m_\gamma + 2) * K_\gamma * \mathscr{N}_{trials} = 80000$. The DSAM system, for a $\mathscr{P}_\gamma$ network, consists of $5 + 3(d_\gamma + m_\gamma) + \frac{1}{2}d_\gamma(d_\gamma + 1) + d_\gamma m_\gamma + \frac{1}{2}m_\gamma(m_\gamma + 1)$ equations, i.e. for the coupled $\mathscr{P}_1$ and $\mathscr{P}_2$ networks system, 89 equations.

- Parameters for $D1MSN$ and $D2MSN$ cells in Striatum, $\mathscr{P}_1$ and $\mathscr{P}_2$.

  Neuromodulatory factors: $\mu = 6.0, \zeta = 0.05$. $C_s^1 = C_s^2 = 1\mu F/cm^2$, $\widehat{g}_{Kir} = \mu 1.2mS/cm^2$, $\widehat{g}_{Ksi} = 0.5mS/cm^2, \widehat{g}_L = 0.3mS/cm^2$. $E_{Kir} = -90mV, E_{Ksi} = -90mV, E_L = -75mV, Es = 20mV$. $V_{h1} = -110mV, V_{c1} = -11mV, V_{h2} = -13.5mV, V_{c2} = 11.8mV$. $T = 310.16K, [Ca]_i = 10pmol/cm^3$, $[Ca]_{ext} = 2\mu mol/cm^3$. $\widehat{P}_{L-Ca} = \mu 4.2nm/s, V_{Ca-h} = -34mV, V_{Ca-c} = 6.1mV$. $\widehat{g}_{Na} = \zeta 120mS/cm^2, \widehat{g}_K = \zeta 10mS/cm^2$, $V_{Na} = 50mV, V_K = -77mV$, $\delta = 20ms, N_{inp} = 500$. $g_{min} = 0.01mS/cm^2$, $g_{max} = 0.05mS/cm^2$.
  $\Gamma_{max} = 0.3, \phi = 0.008$. $T_1 = 2s, T_2 = 1.1s, T_3 = 0.5s$.
  XXXmol ==>? M

- Parameters for cells in the internal and external globus pallidus GPi and GPe, $\mathscr{P}_3$ and $\mathscr{P}_4$.

  $C_s^3 = C_s^4 = 1\mu F/cm^2, g_L = 0.1mS/cm^2, g_{Na} = 120mS/cm^2, g_K = 30mS/cm^2, g_T = 0.5mS/cm^2, g_{Ca} = 120mS/cm^2$, $g_{AHP} = 30mS/cm^2$, $v_L = -55mV, v_{Na} = 55mV, v_K = -80mV, v_{Ca} = 120mV$, $\tau_n^0 = 0.05ms$, $\tau_h^0 = 0.05ms, \tau_n^1 = 0.27ms, \tau_h^1 = 0.27ms, \tau_u = 30ms$. $\sigma_n^\tau = -12mV, \sigma_h^\tau = -12mV, \theta_n^\tau = -40mV$, $\theta_h^\tau = -40mV$. $\sigma_m = 10mV, \sigma_n = 14mV, \sigma_h = -12mV, \sigma_u = -2mV, \sigma_a = 2mV, \sigma_s = 2mV$. $\phi_u = 1, \phi_n = 0.1, \phi_h = 0.05, k_{Ca} = 15XXX, k = 30XXX, \epsilon = 0.0001XXX, \theta_u = -70mV, \theta_m = -37mV, \theta_n = -50mV, \theta_h = -58mV, \theta_s = -35mV, \theta_a = -57mV$,

- Parameters for cells in the subthalamic nucleus STN, $\mathscr{P}_5$.

  $C_s^5 = 1\mu F/cm^2, g_L = 2.25mS/cm^2, g_{Na} = 37.5mS/cm^2, g_K = 45mS/cm^2, g_T = 0.5mS/cm^2, g_{Ca} = 0.5mS/cm^2$, $g_{AHP} = 9mS/cm^2$, $v_L = -60mV, v_{Na} = 55mV, v_K = -80mV, v_{Ca} = 140mV$, $\tau_n^0 = 1ms$, $\tau_h^0 = 1ms, \tau_n^1 = 100ms, \tau_h^1 = 500ms, \tau_u^0 = 7.1ms, \tau_u^1 = 17.5ms$. $\sigma_n^\tau = -26mV, \sigma_h^\tau = -3mV, \theta_n^\tau = -80mV, \sigma_u^\tau = -2.2mV$, $\theta_h^\tau = -57mV$. $\sigma_m = 15mV, \sigma_n = 8mV, \sigma_h = -3.1mV, \sigma_u = -2mV, \sigma_a = 7.8mV, \sigma_s = 8mV, \sigma_b = 0.07$. $\phi_u = 1, \phi_n = 0.1, \phi_h = 0.05, k_{Ca} = 15XX, k = 30XX, \epsilon = 0.0001XX, \theta_u = -67mV, \theta_m = -30mV, \theta_n = -32mV, \theta_h = -39mV, \theta_s = -39mV, \theta_a = -63mV, \theta_b = 0.25, \theta_u^\tau = 68mV$,

- Parameters for DA cells in the substantia nigra pars compacta SNc, $\mathscr{P}_6$.

  The soma and dendritic membranes have the same specific capacitance $C_s^6 = C_d^6 = 1\mu F/cm^2$. $g_{Na} = 3.2mS/cm^2, g_{CaT} = 1.5mS/cm^2, g_{KDRS} = 3.2mS/cm^2, g_{K(Ca)} = 1.2mS/cm^2$, $g_c = 0.1mS/cm^2, g_{KDRD} = 0.14mS/cm^2, g_{NMDA} = 1.25mS/cm^2, g_L = 0.18mS/cm^2$ are conductancesXX, $V_{Na} = 50mV, V_{Ca} = 120mV, V_K = -85mV, q = 12.5mV$, $V_{NMDA} = 0mV$, $V_L = -50mV$ are equilibrium values XX, $K_{Ca} = 0.4\mu M, [Mg^{++}]_0 = 1.4mM, K_{Mg} = 10mM$, $R_{pump} = 18\mu A/cm^2$, $\Gamma([Na], eq) = 8mM$, $\alpha = 0.173mM.cm^2/\mu A.s$,
  $\beta = 0.104\mu M.cm^2/\mu A.s, k_{Ca} = 1s^{-1}$.
  The coupling current parameter between soma and dendrite in DA cells, is $p_{DA} = 0.35$.

- Parameters for FS interneurons in Striatum, $\mathscr{P}_7$ and $\mathscr{P}_8$.

  $C_s^7 = C_s^8 = 1\mu F/cm^2, g_L = 0.25mS/cm^2, V_L = -70mV, g_{Na} = 112.5mS/cm^2, V_{Na} = 50mV, \theta_m = -24.0mV$ <===
  $XXXXX, \sigma_m = 11.5mV, \theta_h = -58."mV, \sigma_h = -6.7mV, \theta_{th} = -60mV, \sigma_{th} = -12mV, g_{Kdr} = 225mS/cm^2, V_K = -90mV, \theta_n = -12.4mV, \sigma_n = 6.8mV, \theta_a = -50mV, \sigma_a = 20mV, \tau_a = 2ms, \theta_b = -70mV, \sigma_b = -6mV, \tau_b = 150ms, g_d = 0.5mS/cm^2$ <==== $XXXXX$.

5 Appendix: The mean field equations.

Let us denote by

$$p_t((Z_i^\alpha)_{i,\alpha}) = p_t((Z_i^1)_{i=1,2,\dots,K_1}, \dots, (Z_j^P)_{j=1,2,\cdots,K_P}) \tag{56}$$

the joint probability distribution of the stochastic variables $(Z_i^\alpha)_{i,\alpha} = (Z_i^\alpha)_{i=1,2,\dots,K_\alpha}^{\alpha=1,2,\dots,P}$.

The Fokker Planck equation for the system (17) is

$$\frac{\partial}{\partial t} p_t((Z_i^\alpha)_{i,\alpha}) = -\sum_{\gamma=1}^{P}\sum_{i=1}^{K_\gamma} \frac{\partial}{\partial Z_i^\gamma}(\mathcal{F}^\gamma(Z_i^\gamma)\, p_t((Z_i^\alpha)_{i,\alpha})) \tag{57}$$

$$-\sum_{\gamma=1}^{P}\sum_{i=1}^{K_\gamma} \frac{\partial}{\partial Z_i^\gamma}\left(\sum_{\alpha=1}^{P}\frac{1}{K_\alpha}\sum_{j=1}^{K_\alpha}\mathscr{M}^{\gamma\alpha}(Z_i^\gamma, Z_j^\alpha)\, p_t((Z_i^\alpha)_{i,\alpha})\right) + \frac{1}{2}\sum_{\gamma=1}^{P}\sum_{i=1}^{K_\gamma}(\beta_V^\gamma)^2 \frac{\partial^2}{\partial (V_i^\gamma)^2} p_t((Z_i^\alpha)_{i,\alpha})$$

We now adopt the Klimontovich approach [13] which has been successfully developed for the kinetic theory of gases and plasmas. The method is here adapted to the derivation of a probabilistic description of the system (17) of noisy interacting spiking neural populations with multicomponent and multicompartmental structure (see also [6]).

One defines the following set variable

$$\mathscr{N}^\mu(U) = \frac{1}{K_\mu}\sum_{i=1}^{K_\mu}\delta(Z_i^\mu - U) \tag{58}$$

where $\delta(\cdot)$ is the Dirac distribution and Z_i^μ is the solution of (18) written for the population $\mathscr{P}_\mu$. Z_i^μ and U are in $\mathbb{R}^{d_\mu+m_\mu+2}$. The expectation value of the stochastic variables $\mathscr{N}^\mu(U)$ with respect to the probability distribution p_t is denoted by $n_t^\mu(U)$, so that

$$n_t^\mu(U) = \langle \mathscr{N}^\mu(U)\rangle_{p_t}. \tag{59}$$

We call $n_t^\mu(U)$ the neural population probability distribution (PPD) for the population $\mathscr{P}_\mu$. In the following, we call Ω the integration space over all variables $Z_l^\delta, l = 1, 2, \dots, K_\delta,\ \delta = 1, 2, \dots, P$. We now derive an equation for $n_t^\mu(U)$. The time derivative $\frac{\partial}{\partial t} n_t^\mu(U)$ is composed of 3 terms

$$\frac{\partial}{\partial t} n_t^\mu(U) = \gamma_1^\mu + \gamma_2^\mu + \gamma_3^\mu. \tag{60}$$

Let us consider each of them separately. The first one, γ_1^μ is given by

$$\gamma_1^\mu = -\int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta \sum_{\gamma=1}^{P}\sum_{i=1}^{K_\gamma}\frac{\partial}{\partial Z_i^\gamma}\left\{\mathcal{F}^\gamma(Z_i^\gamma) p_t((Z_i^\alpha)_{i,\alpha})\right\}\frac{1}{K_\mu}\sum_{j=1}^{K_\mu}\delta(Z_j^\mu - U). \tag{61}$$

Being a probability distribution, p_t has nice vanishing properties for sufficiently large values of the variables Z_i^μ. Thus, a simple integration by parts on (61) enables us to deduce the following expression for γ_1^μ

$$\gamma_1^\mu = \int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta \sum_{\gamma=1}^{P}\sum_{i=1}^{K_\gamma}\left\{\mathcal{F}^\gamma(Z_i^\gamma) p_t((Z_i^\alpha)_{i,\alpha})\right\}\frac{\partial}{\partial Z_i^\gamma}\frac{1}{K_\mu}\sum_{j=1}^{K_\mu}\delta(Z_j^\mu - U). \tag{62}$$

Clearly, $\frac{\partial}{\partial Z_i^\gamma}\sum_{j=1}^{K_\mu}\delta(Z_j^\mu - U) = 0$ if $\gamma \neq \mu$, so that

$$\frac{\partial}{\partial Z_i^\mu}\sum_{j=1}^{K_\mu}\delta(Z_j^\mu - U) = \frac{\partial}{\partial Z_i^\mu}\delta(Z_i^\mu - U), i = 1, 2, \dots, K_\mu.$$

Hence (62) can be rewritten

$$\gamma_1^\mu = \frac{-\partial}{\partial U}\left\{\int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta \sum_{i=1}^{K_\mu}\left\{\mathcal{F}^\mu(U) p_t((Z_i^\alpha)_{i,\alpha})\right\}\frac{1}{K_\mu}\delta(Z_i^\mu - U)\right\} \tag{63}$$

$$= \frac{-\partial}{\partial U}\left\{\int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta \left\{\mathcal{F}^\mu(U)\mathscr{N}^\mu(U) p_t((Z_i^\alpha)_{i,\alpha})\right\}\right\}. \tag{64}$$

Finally, using (59), γ_1^μ is given by

$$\gamma_1^\mu = \frac{-\partial}{\partial U}(\mathcal{F}^\mu(U) n_t^\mu(U)). \tag{65}$$

Let us now consider the second term γ_2^μ in (60)

$$\gamma_2^\mu = -\int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta \sum_{\gamma=1}^{P}\sum_{i=1}^{K_\gamma} \frac{\partial}{\partial Z_i^\gamma}\left\{\sum_{\alpha=1}^{P}\frac{1}{K_\alpha}\sum_{l=1}^{K_\alpha}\mathscr{M}^{\gamma\alpha}(Z_i^\gamma, Z_l^\alpha) p_t((Z_i^\alpha)_{i,\alpha})\right\}\frac{1}{K_\mu}\sum_{j=1}^{K_\mu}\delta(Z_j^\mu - U). \tag{66}$$

By the same argument used for the derivation of (65), an integration by parts in (66) implies

$$\begin{aligned}
\gamma_2^\mu &= \int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta \sum_{\gamma=1}^{P}\sum_{i=1}^{K_\gamma}\left\{\sum_{\alpha=1}^{P}\frac{1}{K_\alpha}\sum_{l=1}^{K_\alpha}\mathscr{M}^{\gamma\alpha}(Z_i^\gamma, Z_l^\alpha) p_t((Z_i^\alpha)_{i,\alpha})\right\}\frac{\partial}{\partial Z_i^\gamma}\frac{1}{K_\mu}\sum_{j=1}^{K_\mu}\delta(Z_j^\mu - U) \\
&= -\frac{\partial}{\partial U}\sum_{\alpha=1}^{P}\int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta \sum_{i=1}^{K_\mu}\frac{1}{K_\alpha}\sum_{l=1}^{K_\alpha}\mathscr{M}^{\mu\alpha}(U, Z_l^\alpha) p_t((Z_i^\alpha)_{i,\alpha})\frac{1}{K_\mu}\delta(Z_i^\mu - U) \\
&= -\frac{\partial}{\partial U}\sum_{\alpha=1}^{P}\int_{R^{d_\alpha+m_\alpha+4}} dU' \int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta \sum_{i=1}^{K_\mu}\frac{1}{K_\alpha}\frac{1}{K_\mu}\sum_{l=1}^{K_\alpha}\mathscr{M}^{\mu\alpha}(U, U')\, p_t((Z_i^\alpha)_{i,\alpha})\delta(Z_i^\mu - U)\delta(Z_l^\alpha - U') \\
&= -\frac{\partial}{\partial U}\sum_{\alpha=1}^{P}\int_{R^{d_\alpha+m_\alpha+4}} dU' \int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta \mathscr{M}^{\mu\alpha}(U, U') p_t((Z_i^\alpha)_{i,\alpha})\mathscr{N}^\mu(U)\mathscr{N}^\alpha(U')
\end{aligned} \tag{67}$$

which again leads to

$$\gamma_2^\mu = -\frac{\partial}{\partial U}\sum_{\alpha=1}^{P}\int_{R^{d_\alpha+m_\alpha+4}} dU' \mathscr{M}^{\mu\alpha}(U, U')\langle \mathscr{N}^\mu(U)\mathscr{N}^\alpha(U')\rangle_{p_t}. \tag{68}$$

The term γ_2^μ has been obtained without the use of approximations and hence is exact. However, it is not really useful in applications. A way to go further consists in making the so called mean field estimate (see Ref)

$$\langle \mathscr{N}^\mu(U)\mathscr{N}^\alpha(U')\rangle_{p_t} \approx \langle \mathscr{N}^\mu(U)\rangle_{p_t}\langle \mathscr{N}^\alpha(U')\rangle_{p_t}. \tag{69}$$

This approximation is valid because the fluctuations of $\mathscr{N}^\mu(U)$ (resp. $\mathscr{N}^\alpha(U')$) are small for K_μ (resp. K_α) large and are $O(\frac{1}{\sqrt{K_\mu}})$ (resp. $O(\frac{1}{\sqrt{K_\alpha}})$).

Accordingly, the coupling term γ_2^μ takes the form :

$$\gamma_2^\mu = -\frac{\partial}{\partial U}\sum_{\alpha=1}^{P}\int_{R^{d_\alpha+m_\alpha+4}} dU' \mathscr{M}^{\mu\alpha}(U, U') n_t^\mu(U) n_t^\alpha(U'). \tag{70}$$

The last diffusive term γ_3^μ can be derived in a similar fashion. It is given by

$$\gamma_3^\mu = \frac{1}{2}\int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta \sum_{\gamma=1}^{P}\sum_{i=1}^{K_\gamma}(\beta_V^\gamma)^2\frac{\partial^2}{\partial(V_i^\gamma)^2} p_t((Z_i^\alpha)_{i,\alpha})\frac{1}{K_\mu}\sum_{j=1}^{K_\mu}\delta(Z_j^\mu - U). \tag{71}$$

Integration by parts gives

$$\gamma_3^\mu = \frac{1}{2}\int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta \sum_{\gamma=1}^{P}\sum_{i=1}^{K_\gamma}(\beta_V^\gamma)^2 p_t((Z_i^\alpha)_{i,\alpha})\frac{1}{K_\mu}\frac{\partial^2}{\partial(V_i^\gamma)^2}\sum_{j=1}^{K_\mu}\delta(Z_j^\mu - U). \tag{72}$$

Here also, one has

$$\frac{\partial^2}{\partial(V_i^\gamma)^2}\sum_{j=1}^{K_\mu}\delta(Z_j^\mu - U) = 0 \text{ if } \gamma \neq \mu \tag{73}$$

while

$$\frac{\partial^2}{\partial(V_i^\mu)^2}\sum_{j=1}^{K_\mu}\delta(Z_j^\mu - U) = \frac{\partial^2}{\partial(V_i^\mu)^2}\delta(Z_i^\mu - U). \tag{74}$$

Thus

$$\gamma_3^\mu = \frac{1}{2}\int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta p_t((Z_i^\alpha)_{i,\alpha})\frac{1}{K_\mu}\sum_{i=1}^{K_\mu}(\beta_V^\mu)^2\frac{\partial^2}{\partial(V_i^\mu)^2}\delta(Z_i^\mu - U). \tag{75}$$

So

$$\gamma_3^\mu = \frac{(\beta_V^\mu)^2}{2}\frac{\partial^2}{\partial v^2}\int_\Omega \prod_{\delta=1}^{P}\prod_{l=1}^{K_\delta} dZ_l^\delta p_t((Z_i^\alpha)_{i,\alpha})\frac{1}{K_\mu}\sum_{i=1}^{K_\mu}\delta(Z_i^\mu - U) \tag{76}$$

and hence

$$\gamma_3^\mu = \frac{(\beta_V^\mu)^2}{2}\frac{\partial^2}{\partial v^2}n_t^\mu(U). \tag{77}$$

6 Conclusion

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